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*Corresponding author
Maher A. Nawkhass
maher.nawkhass@su.edu.krd

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Revised Harmonious Fuzzy Technique for Solving Fully Fuzzy Multi-Objective Linear Fractional Programming Problems

Maher A. Nawkhass*

Department of Mathematics, College of Education, Salahaddin University-Erbil, Erbil, Kurdistan Region, Iraq.

ABSTRACT

The revised harmonious fuzzy technique (RHFT) is a method used to solve fuzzy optimization problems. It was capitalized as an extension of the classical linear programming technique to handle constraints and objectives that are fuzzy. The harmonious fuzzy technique HFT aims to find a solution that satisfies the uncertain restraints and optimizes the uncertain objectives while taking into account the uncertainty or fuzziness of the problem parameters. This work demonstrates how the RHFT can be utilized to dexterously solve “fully fuzzy multi-goal linear fractional programming (FFMOLFP) problems”. Initially, the FFMOLFP problem can be converted to “single goal linear fractional programming (SOLFP) problems” consuming the modified brittle linear technique. Second, the RHFT is applied to converted brittle problems into linear programming problem, which follow, “the single-goal problem” is made on so on applied the revised harmonious fuzzy for apiece level**.** at the end, the obtained LPP will be solved by applied the simplex algorithm. To illustrate the application of this method, two examples will be provided. Also, the numerical results are simulated by comparing between proposed method and efficient ranking function methods for fully fuzzy linear fractional programming problems FFLFPP

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1.Introduction

In recent years, multi-goal linear fractional programming MOLFPP has acquired important interest due to its aptitude to model real-world decision-making problems with conflicting objectives. The uncertain set theory has also been extensively applied to handle uncertainties and vagueness in decision-making processes. Thus, the integration of fuzzy logic and "multi-goal linear fractional programming" has led to the development of FFMOLFP models. While existing methods have made significant contributions to solving FFMOLFP problems, there is still area for improvement in terms of solution quality and computational efficiency. In this context, this article proposes a revised harmonious fuzzy technique for solving FFMOLFP.

(Das et al., 2017) their research focuses on devising a very effective solution to solve "the completely FLFPP. In order to achieve this aim, we develop a novel approach by combining the Charnes-Cooper practice with MOLPP. (Pramy, 2018) an endeavor has been undertaken to develop a method for determining the FMOLFP issue. Initially, the FMOLFP issue is transformed into a MOLFPP using the graded mean integration representation (GMIR) approach developed by (Chen and Hsieh, 1999). All the imprecise parameters of the FMOLFP issue are transformed into precise values. Subsequently, the MOLFP challenge is converted into a solitary goal LP problem using a proposition provided by Nuran Guzel. (Ammar and Khalifa, 2019) their work focuses on solving a problem known as "multi-objective linear fractional programming (interval-valued fuzzy IVF MOLFP)", which involves fuzzy limiting factor in both the goal functions and restraints. Instead of using regular fuzzy numbers, these fuzzy parameters are described by interval-valued fuzzy facts denoted as (g, d). (Loganathan and Ganesan, 2019) provide a method for resolving completely "fuzzy linear fractional programming problems", where all strictures and variables are represented as triangular fuzzy integers. By converting all the triangular fuzzy numbers into their parametric form, we may transform the fractional programming issue into a single goal linear programming problem in parametric form. By using novel "fuzzy arithmetic and fuzzy ranking techniques", we are able to get the finest solution for the given totally FLPP matter without the need to transform it into its corresponding crisp LPP.

(Van Hop, 2020) identify all the corresponding connections between two imprecise numbers. Instead of utilizing absolute value to express the fuzzy number, they propose the use of new relative procedures to comparison two fuzzy numbers. These criteria pertain to the prevailing level at which one fuzzy number surpasses the extra in terms of its location and form. "The absolute fuzzy dominant grade beside comparative fuzzy dominant grade" are devised to quantify the disparities among two fuzzy numbers in the context of various constraint kinds. These measurements have the ability to include all the distinct features and relative arrangements of fuzzy numbers. (Suleiman and Nawkhass, 2022) they propose the use of the geometric arithmetic mean as a solution method for non-LPP. In order to do this, they will use the classical optimization theorem to deduce the inequality. This will include establishing the requisite and definitive criteria for finding the motionless arguments of the general disparity restraint optimization issues. (Loganathan and Ganesan, 2022) presents a modified version of the Gauss Elimination Approach (GEA) that is specifically designed to solve completely FFMOLFPP. The method is tailored to handle problems involving triangular fuzzy numbers (TFNs). The completely "fuzzy linear fractional programming" issue is first transformed into an analogous FFLPP by appropriate conversion techniques. Subsequently, the optimal value of each objective function is determined separately, while seeing the same set of constraints. (Azeez and Hamadameen, 2024) proposed method utilizes the concepts of alpha-cut technique with truth degrees technique on probability distribution, linear fuzzy membership function (LFMF), linear fuzzy ranking function (LFRF), trapezoidal fuzzy number, triangular fuzzy number and expectation weighted summation (EWS) technique.(Ali and Yaba, 2023) their research proposes a new method of automatically segmenting red blood cells from microscopic blood smear images by using fuzzy approach. The study suggests a novel combination of image processing techniques and extensive preprocessing to achieve superior segmentation performance/

(Nawkhass and Sulaiman, 2022a) their study presents a technique for converting and solving a problem by changing "the symmetric fuzzy approach". It proposes a procedure and demonstrates how FLFPP may be solved deprived of increasing the computational complexity. Additionally, it presents a method that employs an optimum average to transform MOLFPP into a singular LFPP via the modification of "the symmetric fuzzy approach". (Nawkhass and Sulaiman, 2022b) present a method for converting and solving the issue applied a "symmetric fuzzy approach". Propose an approach and demonstrate how fuzzy LFPP may be resolved deprived of increasing computational power. The purpose of our approach is to transform a CMOLFPP into a LPP by using a SFLFP. To showcase the effectiveness of the proposed method.

The proposed method utilizes uncertain set theory and harmonious search methods to enhance the problem-solving approach, for FFMOLFP challenges. The refined harmonious fuzzy approach aims to generate a set of nearly optimal solutions while considering the uncertainties and vagueness associated with decision variables and objective functions. This method strives to strike a balance between goals. A key improvement of the fuzzy approach lies in its ability to effectively tackle FFMOLFP complexities. While previous methods have mainly focused on imprecise or interval based formulations the recommended strategy allows for the transformation of decision variables and objective functions, into fuzzy representations. This advancement facilitates a accurate treatment of uncertainty resulting in more robust and reliable solutions.

This piece is structured into twelve sections each delving into an exploration of the revised harmonic fuzzy technique (RHFT), for addressing FFMOLFPP. The second section lays out the concepts of triangular numbers and the harmonic mean method, essential for grasping subsequent sections. Moving on to section 3 we encounter the introduction of fuzzy harmonic mean as a component of RHFT. Section 4 delves into the methodology of the fuzzy technique elucidating the key steps in transforming a complex FMOLFPP problem into a series of solvable LPPs. Section 5 illustrates the application of this approach in tackling FFMOLFPP through real world examples. In section 6 a detailed explanation is provided on solving LFPP using the recommended method equipping readers with the proficiency to implement RHFT effectively. Sections 7 present procedures for addressing FFMOLFPP and offer aids like flowcharts in section 8 to enhance reader comprehension. Numerical examples are included in section 9 to illustrate RHFT implementation and underscore its relevance. In section 10 simulate the numerical results. Section 11 addressed by results and discussion. Lastly section 12 outlines findings while suggesting areas for further exploration aiming to equip readers with a comprehensive understanding of RHFTs role, in enhancing decision making under challenging circumstances.

2.Preliminaries

This section gives necessary notations, theorem and remark related to this work.

Definition 2.1. (MAHDAVI et al., 2009) if X is a group of objects represented basically by x, then a fuzzy set A in X is demarcated to be a set of well-ordered couples $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x)$ is entitled the membership function for the uncertain set. The membership function maps apiece component of X to a membership value among 0, 1. We assume that X is the real line R.

Definition 2.2. (Pandian and Jayalakshmi, 2010) A fuzzy number $\tilde{a}$ is a triangular fuzzy number represented by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{a^{\sim}}(x)$ is assumed as follow:

$$\mu_{a^{\iota}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for} \quad a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for} \quad a_2 \le x \le a_3 \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$$

Definition 2.3. (Pandian and Jayalakshmi, 2010) Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

(i) $(a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii) $(a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$, for $k \ge 0$.

(iv) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$, for $k < 0$

Let $F(R)$ be the set of all real triangular uncertain numbers.

Definition 2.4. (Pandian and Jayalakshmi, 2010) Assume $\tilde{A} = (a_1, a_2, a_3)$ be in $F(R)$. Then

(i) $\tilde{A} = \tilde{B} \Leftrightarrow a i = b i$, for all for i

(ii) $\tilde{A} \le \tilde{B} \Leftrightarrow a i \le b i$, for all for $i = 1$ to 3.

Definition 2.5. (Xu, 2009) Let wight harmonic mean (WHM) defined by: $(R^+)^n \to R^+$, if

$$WHM(a_1, a_2, \dots, a_n) = \frac{1}{\sum_{j=1}^{n} \frac{w_j}{a_j}}$$

where $a_j (j = 1,2, \dots, n)$ is a group of positive real numbers, $w_j = (w_1, w_2, ..., w_n)^T$ is the weight course of $a_j (j = 1,2, \dots, n)$, beside $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1 \cdot R^+$ is the set of all positive real numbers. Specially, if $w_i = 1, w_j = 0, \forall j \neq i$, then WHM $(a_1, a_2, \dots, a_n) = a_j$; if $w = (1/n, 1/n, \dots, 1/n)^T$, then the WHM is concentrated to the harmonic mean (HM):

$$HM(a_1, a_2, \dots, a_n) = \frac{n}{\sum_{j=1}^{n} \frac{1}{a_j}}$$

2.1 Basic Concepts of Goal Programming (GP)

Goal Programming is a powerful mathematical optimization technique designed to handle problems with multiple, often conflicting, objectives. Unlike traditional linear programming, which seeks a single optimal solution, Goal Programming aims to find a solution that satisfies or comes as close as possible to achieving a set of desired goals.

2.1.1 Core Concepts of Goal Programming: can be explained as following:

2.1.1.1 Goals, Not Just Objectives:

- In traditional optimization, we have objective functions that we want to maximize or minimize.
- In GP, we have goals, which represent target levels of achievement we hope to reach for various objectives. These goals can be desired levels of profit, production, cost, or other key performance indicators.

2.1.1.2 Types of Goal Programming:

Weighted goal programming type is the most common type. It assigns weights to each deviational variable, reflecting the relative importance of achieving each goal.

3 Fuzzy Harmonic Mean

If $\hat{a}$ is defined as the set of values $\hat{a} = [a^L, a^M, a^U]$, where a^U is more than a^M and a^L is greater than zero, then $\hat{a}$ is mentioned to as a triangular fuzzy number. The inferior and higher values of $\hat{a}$ are represented by a^L and a^U, respectively, while the modal value is represented by a^M. To be more specific, if any two of the elements a^L, a^M, and a^U are equivalent, then $\hat{a}$ is abridged to an intermission number. Contrariwise, if all of the elements a^L, a^M, and a^U are equivalent, beside that $\hat{a}$ is reduced to a real number. The set of all triangular uncertain numbers is represented by the symbol Ω for the sake of simplicity. The following is an introduction to certain operational laws of triangular fuzzy numbers, which we will first provide below:

Definition 3.1. (Xu, 2009) Let $\hat{a} = [a^L, a^M, a^U]$ and $\hat{b} = [b^L, b^M, b^U]$ be two triangular fuzzy numbers, then the basic arithmetic operations can be presented as follows:

(1) $\hat{a} + \hat{b} = [a^L, a^M, a^U] + [b^L, b^M, b^U] = [a^L + b^L, a^M + b^M, a^U + b^U]$.

(2) $\lambda\hat{a} = \lambda[a^L, a^M, a^U] = [\lambda a^L, \lambda a^M, \lambda a^U]$, where $\lambda > 0$.

(3) $1/\hat{a} = 1/[a^L, a^M, a^U] = [1/a^U, 1/a^M, 1/a^L]$.

(4) $\hat{a} * \hat{b} = [a^L, a^M, a^U] * [b^L, b^M, b^U] = [a^L * b^L, a^M * b^M, a^U * b^U]$.

4 Methodology

4.1. Formulation of the FFLFP Problems.

Let have the FFLFP problems consider as following:

Max. (or Min.) $\tilde{Z} = \frac{(\tilde{C}_1^t \otimes \tilde{X}_i)}{(\tilde{C}_2^t \otimes \tilde{X}_i)}$

subject to

$\tilde{A} \otimes \tilde{X}_i = \tilde{b}_i \tilde{X}_i$: is non- negative fuzzy number,

where $\tilde{X}_i = [\tilde{X}_j]_{n\times 1}$ i = 1,2 , $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b}_i = [\tilde{b}_i]_{m\times 1}$, i = 1,2.

and $\tilde{a}_{ij}, \tilde{C}_j, \tilde{X}_j, \tilde{b}_i \in F(R)$ for $\forall j$, $j = 1,2$ and geater than zero. (1)

4.2. Formulation of the FFMOLFP Problems.

Assume FFMOLFP problems consider as following:

Max. (or Min.)$\tilde{Z} = \frac{(\tilde{C}_{11}^t \otimes \tilde{X}_i)}{(\tilde{C}_{12}^t \otimes \tilde{X}_i)}, \frac{(\tilde{C}_{21}^t \otimes \tilde{X}_i)}{(\tilde{C}_{22}^t \otimes \tilde{X}_i)}, \dots, \frac{(\tilde{C}_{n1}^t \otimes \tilde{X}_i)}{(\tilde{C}_{n2}^t \otimes \tilde{X}_i)}$

subject to

$\tilde{A} \otimes \tilde{X}_i = \tilde{b}_i$

$\tilde{X}_i$: is non- negative fuzzy number.

where $\tilde{C}_i^t = [\tilde{C}_j]_{1\times n}$ for all i, j = 1, …, n, $\tilde{X}_i = [\tilde{X}_i]_{n\times 1}$ for all i, j = 1, …, n

$\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b}_i = [\tilde{b}_i]_{m\times 1}$ and $\tilde{a}_{ij}, \tilde{C}_j, \tilde{X}_i, \tilde{b}_i \in F(R)$ and geater than zero. (2)

4.3 Revised Harmonious Fuzzy method

Here we explain the basic ideas and the main formula used as follows:

4.3.1 Main Formula of the Revised Harmonious Fuzzy (RHF) Method

Let us consider the following RHF method:

First we gave two cases:

Case 1: RHF_l problems for maximum where l for lower case, as follow

$$\left.\begin{array}{l} \text{Max. } Z_{RHF_l} = [2 * (\tilde{C}_{1l}^{t} \otimes \tilde{X}_1 \oplus d_l)] * [2 * (\tilde{C}_{2l}^{t} \otimes \tilde{X}_2 \oplus f_l) / (\tilde{C}_{1l}^{t} \otimes \tilde{X}_1 \oplus d_l) \oplus (\tilde{C}_{2l}^{t} \otimes \tilde{X}_2 \oplus f_l)] \\ \text{subject to} \\ \tilde{A}_l \otimes \tilde{X}_i = \tilde{b}_l \\ \tilde{X}\tilde{X}_i : \text{is non- negative fuzzy number,} \\ \text{where } \tilde{C}_i^t = [\tilde{C}_j]_{1\times n}\ i = 1,2, \tilde{X}_i = [\tilde{X}_i]_{n\times 1}\ i = 1,2,\ \tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_i]_{m\times 1}, \\ d, f \geq 0 \text{ are fuzzy constants and } \tilde{a}_{ij}, \tilde{C}_j, \tilde{X}_i, \tilde{b}_i \in F(R) \text{ and geater than zero.} \end{array}\right\} \quad (3)$$

Case 2: RHF_u problems for minimum where u for upper case, as follow

$$\left.\begin{array}{l} \text{Max. } Z_{RHF_u} = [2 * (\tilde{C}_{1u}^{t} \otimes \tilde{X}_1 \oplus d_u)] * [2 * (\tilde{C}_{2u}^{t} \otimes \tilde{X}_2 \oplus f_u) / (\tilde{C}_{1u}^{t} \otimes \tilde{X}_1 \oplus d_u) \oplus (\tilde{C}_{2u}^{t} \otimes \tilde{X}_2 \oplus f_u)] \\ \text{subject to} \\ \tilde{A}_u \otimes \tilde{X}_i = \tilde{b}_u \\ \tilde{X}_i : \text{is non- negative fuzzy number,} \\ \text{where } \tilde{C}_i^t = [\tilde{C}_j]_{1\times n}\ i = 1,2, \tilde{X}_i = [\tilde{X}_i]_{n\times 1}\ i = 1,2,\ \tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_i]_{m\times 1}, d, f \geq 0 \text{ are} \\ \text{fuzzy constants and } \tilde{a}_{ij}, \tilde{C}_j, \tilde{X}_i, \tilde{b}_i \in F(R). \text{ and greater than zero.} \end{array}\right\} \quad (4)$$

5. The Proposed Method to Converts and Solve FFMOLFP Problems by Revised Harmonious Fuzzy Method.

For the purpose of this investigation, we suggested a novel approach to transform FFMOLFPP into a FFOLFPP. As a result, we were able to obtain the best optimal fuzzy solution.

Firstly, the weighting problem of FFMOLFPP is transformed to the FFOLFPP.

The weighting problem of FFMOLFPP:

$$\left.\begin{array}{l} \text{Max. (or Min.) } \tilde{Z} = \frac{\sum_{n=1}^{k} w^n(\tilde{C}_{11n}^{t} \otimes \tilde{X})}{\sum_{n=1}^{k} w^n(\tilde{C}_{21n}^{t} \otimes \tilde{X}_i)}, \frac{\sum_{n=1}^{k} w^n(\tilde{C}_{12n}^{t} \otimes \tilde{X}_i)}{\sum_{n=1}^{k} w^n(\tilde{C}_{22n}^{t} \otimes \tilde{X}_i)}, \dots, \frac{\sum_{n=1}^{k} w^n\left(\tilde{C}_{1jn}^{t} \otimes \tilde{\tilde{X}}_i\right)}{\sum_{n=1}^{k} w^n\left(\tilde{C}_{2jn}^{t} \otimes \tilde{X}_i\right)}, \text{ for all } j = 1, \dots, n \\ = \frac{w^1\left((\tilde{c}_{11\,1})^t \otimes \tilde{\tilde{X}}_i\right) \oplus w^2((\tilde{c}_{11\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k((\tilde{c}_{11\,k})^t \otimes \tilde{X}_i)}{w^1((\tilde{c}_{21\,1})^t \otimes \tilde{X}_i) \oplus w^2((\tilde{c}_{21\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k((\tilde{c}_{21\,k})^t \otimes \tilde{X}_i)}, \frac{w^1((\tilde{c}_{12\,1})^t \otimes \tilde{X}_i) \oplus w^2((\tilde{c}_{12\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k((\tilde{c}_{12\,k})^t \otimes \tilde{X}_i)}{w^1((\tilde{c}_{22\,1})^t \otimes \tilde{X}_i) \oplus w^2((\tilde{c}_{22\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k((\tilde{c}_{22\,k})^t \otimes \tilde{X}_i)}, \dots, \\ \frac{w^1((\tilde{c}_{1n\,k})^t \otimes \tilde{X}_i) \oplus w^2((\tilde{c}_{1n\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k((\tilde{c}_{1n\,k})^t \otimes \tilde{X}_i)}{w^1((\tilde{c}_{2n\,k})^t \otimes \tilde{X}_i) \oplus w^2((\tilde{c}_{2n\,2})^t \otimes \tilde{X}_i) \oplus \dots \oplus w^k\left((\tilde{c}_{2n\,k})^t \otimes \tilde{\tilde{X}}_i\right)}. \end{array}\right\} \quad (5)$$

Subject to

$\tilde{A} \otimes \tilde{X}_i = \tilde{b}_i$

$\tilde{X}_i \geq 0,\ \sum_{r=1}^{k} w^r = 1,\ w^r \geq 0.$

Secondly, the FFMOLFPP is transformed to the FFOLFPP:

Max. (or Min.) $\tilde{Z} = \frac{(\tilde{C}_1^t \otimes \tilde{X})}{(\tilde{C}_2^t \otimes \tilde{X})}$,

Subject to

$\tilde{A} \otimes \tilde{X} = \tilde{b}, \tilde{X} \geq 0.$

Thirdly, construct Max. Z_{RHF_l} problem for the given (FOLFP) problem.

Finally, solve the Max. Z_{RHF_l} problem using Simplex method for first term and use proposed method in section 4 for second term to find the optimal solution.

6 Proposed Method to Solve Linear Fractional Programming Problems

The technique applied the simplex algorithm for solving a LFPP. Consider the following problem:

$$\left.\begin{array}{ll} \text{Minimize} & \frac{p^t x+\alpha}{q^t x+\beta} \\ \text{subject to} & Ax \le b \\ & x \ge 0 \end{array}\right\} \quad (6)$$

Where p and q are n-vectors, b is m-vector, A is $m \times n$ matrix and α and β are scalars.

Let us assume that the set S, which is defined as $x: Ax \le b$ and $x \ge 0$, is compact, that's mean satisfy three point 1) the transformed feasible set being effectively bounded. 2) implying the existence solution. 3) Specific structure condition.

Furthermore, let us assume that the value of $q^t\, x + \beta$ is greater than zero for every x that belongs to S. The following linear programming (LP) is obtained by letting z equal to $1/(q^t\, x+\beta)$ and y equal to zx, and then multiplying the restraints $Ax \le b$ by z.

$$\left.\begin{array}{ll} \text{Minimize} & \alpha z + p^t y \\ \text{subject to} & Ay - bz \le 0 \\ & \beta z + q^t y = 1 \\ & y \ge 0 \\ & z \ge 0 \end{array}\right\} \quad (7)$$

To begin, it is important to keep in mind that if (y, z) is a workable solution to the problem described above, then z is greater than zero. Given that if z is equal to zero, then y must be such that Ay is less than zero and y is more than zero. This implies that y is a direction of S, which is contrary to the compactness assumption. Given that $(\bar{y}, \bar{z})$ is an ideal solution to the LP mentioned before, we are now able to establish that $\bar{x} = \bar{y}/\bar{z}$ is also an ideal solution to the fractional program presented earlier.

It is important to observe that the inequality $A\bar{x} \le b$ holds, and also that $\bar{x} \ge 0$. This implies that $\bar{x}$ is a valid solution to the fractional program. In order to demonstrate the optimality of $\bar{x}$, consider x to be a solution that satisfies the conditions $Ax \le b$ and $x \ge 0$. It should be noted that the inequality $q^t\, x + \beta > 0$ is assumed, and that the vector (y, z) is a valid solution to the LP, where $y = x/(q^t\, x+\beta)$ and $z = 1/(q^t\, x+\beta)$. Given that $(\bar{y}, \bar{z})$ is an optimum solution to the linear program, it follows that $p^t\bar{y} + \alpha\bar{z}$ is less than or equal to $p^t y + \alpha z$. By replacing the variables $\bar{y}$, y, and z, the inequality may be expressed as $\bar{z}(p^t\bar{x}+\alpha) \le (p^t\, x+\alpha)/(q^t\, x+\beta)$. The result may be obtained by partitioning the left-hand side by $1 = q^t\bar{y} + \beta\bar{z}$

If $q^t\, x + \beta$ is negative for every $x \in S$, then by defining -z as the reciprocal of $(q^t\, x + \beta)$ and y as the product of z and x, we get the LP as follow:

$$\left.\begin{array}{l} \text{Minimize } -\alpha z - p^t y \\ \quad \text{subject to} \\ \quad Ay - bz \le 0 \\ \quad -\beta z - q^t y = 1 \\ \quad y \ge 0 \\ \quad z \ge 0 \end{array}\right\} \quad (8)$$

In a manner that is comparable to the previous statement, if the linear program $(\bar{y}, \bar{z})$ is solved, then the fractional programming issue may be solved by multiplying $\bar{x}$ by $\bar{y}$ divided by $\bar{z}$.

7. Proposed algorithm for Solving Fully Fuzzy Multi-Objective Linear Fractional Programming

Problems

This algorithm utilizes the revised harmonious fuzzy technique (RHFT) to solve FFMOLFP.

Step 1: Convert the FFMOLFP problem to a fully fuzzy linear fractional programming (FFLFP) problem to get the formula 5

Step 2: Convert the FFLFP problem to a revised harmonious fuzzy (RHF) problem.

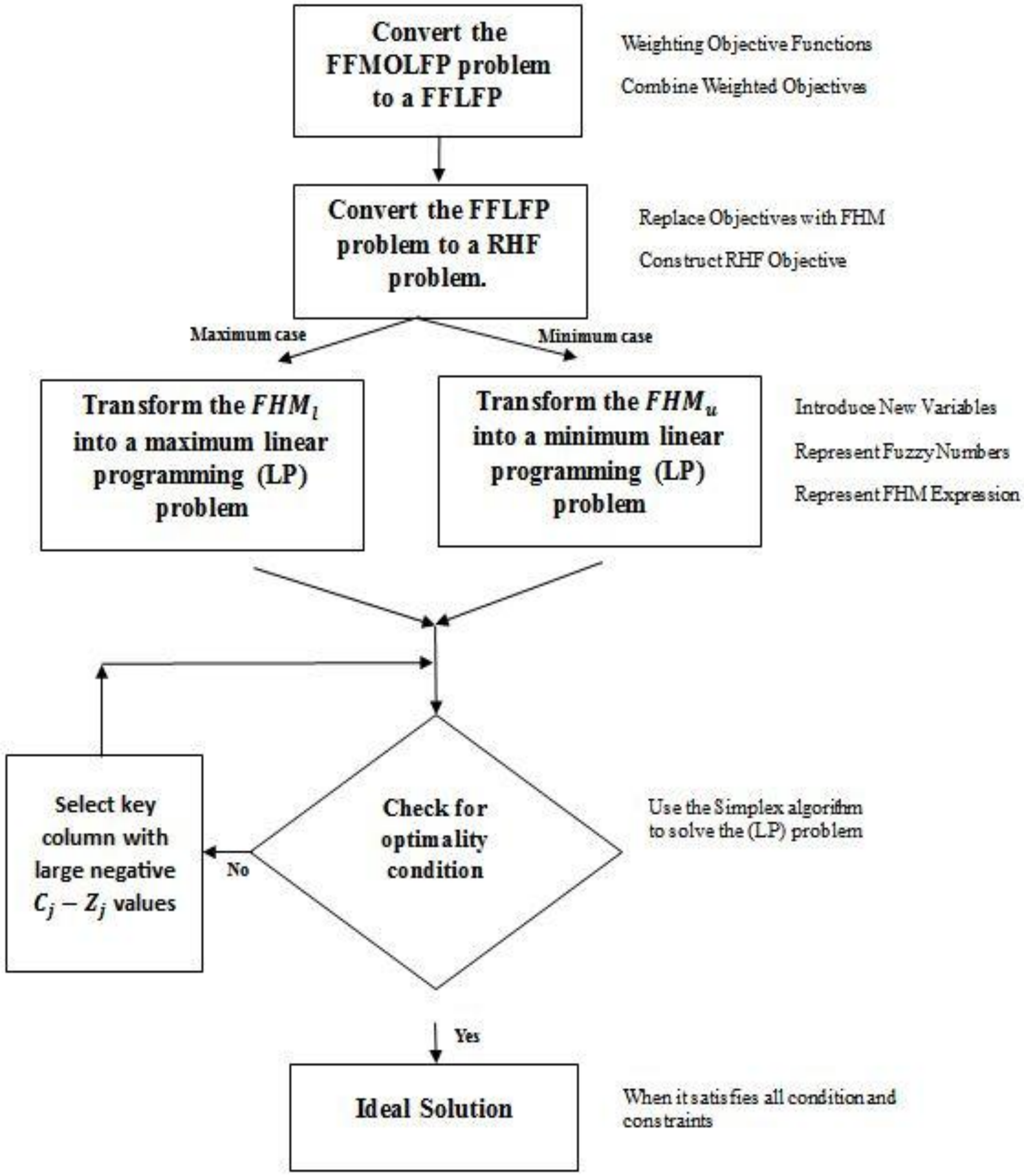
Note: We have two scenarios, for Max; when the case FFLFP changes to FHM_l and when the case FFLFP changes to FHM_u.

Step 3: Transform the FHM_l issue into a (LP) problem.

Step 4: Solve the Max. Z_{RHF_l} problem using Simplex method for first term and use proposed method in section 4 for second term to find the optimal solution.

8. Flowchart of Proposed algorithm for Solving Fully FFMOLFPP

Here is a visual diagram that outlines the steps of the suggested approach, for addressing FFMOLFP. This diagram illustrates the processes and strategies utilized in the proposed technique aiding in comprehension and implementation of the algorithm. It serves as a tool to improve clarity and facilitate navigation through the algorithm.



Here is a visual diagram that illustrates the suggested method, for addressing FFMOLFP concerns. It helps illustrate the step-by-step process involved in the conversion and solution of the problem.

9. Numerical Examples

Using an issue that really exists in the world, we will demonstrate the suggested method in this section. It is clear that the LFPP is an unpredictable optimization problematic because of the fluctuations in the maximum daily needs that it presents. It is thus impossible to predict how much of any product or component will be used. For this reason, we shall treat the issue as a FLFP problem. In order to account for any unknown value, we use triangular fuzzy numbers. The solution that has been given will also be successful in solving the mathematical programming challenge.

Problem 1: (Production Planning) (Das et al., 2017)

Phoenix Woodworks, located in Phoenix, Arizona, is a successful furniture manufacturer that focuses on producing two specific types of products: handcrafted tables (Product A) and sophisticated chairs (Product B). Product A generates a profit of approximately \$4, \$5, or \$8 per unit, while Product B generates a profit of approximately \$2, \$3, or \$7 per unit. First, the cost for each unit of the above items is around \$2, \$3, and \$6, respectively, and approximately \$1, \$2, and \$5, respectively. secondly, the yields profit of around \$2, \$6, and \$9 per unit, and approximately \$3, \$4, and \$6 per unit, correspondingly. The cost for each unit of the above items is around \$1, \$4, or \$7, and approximately \$2, \$3, or \$5, correspondingly. Thirdly the yields profits of around \$3, \$6, and \$7 per unit, and approximately \$2, \$3, and \$5 per unit, respectively. The cost for each unit of the aforementioned items is around \$2, \$4, or \$5, and approximately \$1, \$2, or \$3, respectively. Assuming that the amount of raw material required to production product A and B is about (2,5,8) units per pound and (3,6,10) units per pound correspondingly, the available supply of this raw material is limited to approximately (5,12,18) pounds. The labor required to produce one unit of product A is about 4, 6, or 7 hours, whereas for product B it is approximately 2, 4, or 8 hours per unit. However, the total amount of labor available every day is approximately 4, 10, or 14 hours. The company's objective is to optimize profitability via smart resource allocation.

Solution:

Let $\tilde{x}_1$ and $\tilde{x}_2$ to be the amount of units of A and B respectively to be produced. Then the above problematic can be expressed as:

$$\text{Maximize } Z = \frac{(4,5,8)\otimes\tilde{x}_1 \oplus (2,3,7)\otimes\tilde{x}_2}{(2,3,6)\otimes\tilde{x}_1 \oplus (1,2,5)\otimes\tilde{x}_2}, \frac{(2,6,9)\otimes\tilde{x}_1 \oplus (3,4,6)\otimes\tilde{x}_2}{(1,4,7)\otimes\tilde{x}_1 \oplus (2,3,5)\otimes\tilde{x}_2}, \frac{(3,6,7)\otimes\tilde{x}_1 \oplus (2,3,5)\otimes\tilde{x}_2}{(2,4,5)\otimes\tilde{x}_1 \oplus (1,2,3)\otimes\tilde{x}_2}$$

subject to:

$$(2,5,8)\otimes\tilde{x}_1 \oplus (3,6,10)\otimes\tilde{x}_2 \le (5,12,18)$$
$$(4,6,7)\otimes\tilde{x}_1 \oplus (2,4,8)\otimes\tilde{x}_2 \le (4,10,14)$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

By step 1 point 1: let $w = (0.5,0.5)$, then obtain to FFMOLFP problem can be printed as:

$$\text{Maximize } Z = \frac{(2,2.5,4)\otimes\tilde{x}_1 \oplus (1,1.5,3.5)\otimes\tilde{x}_2}{(1,1.5,3)\otimes\tilde{x}_1 \oplus (0.5,1,2.5)\otimes\tilde{x}_2}, \frac{(1,3,4.5)\otimes\tilde{x}_1 \oplus (1.5,2,3)\otimes\tilde{x}_2}{(0.5,2,3.5)\otimes\tilde{x}_1 \oplus (1,1.5,2.5)\otimes\tilde{x}_2}, \frac{(1.5,3,3.5)\otimes\tilde{x}_1 \oplus (1,1.5,2.5)\otimes\tilde{x}_2}{(1,2,2.5)\otimes\tilde{x}_1 \oplus (0.5,1,1.5)\otimes\tilde{x}_2}$$

Therefore, we get FFLFP programming by using step 1 point 2 & 3

$$\text{Maximize } Z = \frac{(4.5,8.5,12)\otimes\tilde{x}_1 \oplus (3.5,5,9)\otimes\tilde{x}_2}{(2.5,5.5,9)\otimes\tilde{x}_1 \oplus (2,3.5,6.5)\otimes\tilde{x}_2},$$

subject to:

$$(2,5,8)\otimes\tilde{x}_1 \oplus (3,6,10)\otimes\tilde{x}_2 \le (5,12,18)$$
$$(4,6,7)\otimes\tilde{x}_1 \oplus (2,4,8)\otimes\tilde{x}_2 \le (4,10,14)$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

By using step 2 we convert the FFLFP problem to a revised harmonious fuzzy (RHF_l) problem as follows:

$$\text{Max. } Z_{RHF_l} = 2 * [(4.5x_1 + 3.5x_2) * (2.5x_1 + 2x_2)]/[(4.5x_1 + 3.5x_2) + (2.5x_1 + 2x_2)]$$

subject to:

$$2x_1 + 3\,x_2 \le 5$$
$$4x_1 + 2\,x_2 \le 4$$
$$x_1, x_2 \ge 0$$

By using step 3 with section 4 (proposed method) and step 4 we get optimal solution as following:
After simplify we have the follow:

$\text{Max. } Z_{RHF_l} = \text{Max. } Z_{RHF_{l1}} * \quad \text{Max. } Z_{RHF_{l2}}$

Subject to:

$2x_1 + 3\,x_2 \le 5$
$4x_1 + 2\,x_2 \le 4$
$x_1, x_2 \ge 0$

where

$\text{Max. } Z_{RHF_{l1}} = 9x_1 + 7\,x_2$

subject to:

$2x_1 + 3\,x_2 \le 5$
$4x_1 + 2\,x_2 \le 4$
$x_1, x_2 \ge 0$

Applied simplex algorithm to solve the problem, we have $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{2}$, $\text{Max. } Z_{RHF_{l1}} = \frac{51}{4}$

$\text{Max. } Z_{RHF_{l2}} = \frac{5x_1 + 4\,x_2}{4.5x_1 + 3.5x_2 + 2.5x_1 + 2x_2}$

subject to:

$2x_1 + 3\,x_2 \le 5$
$4x_1 + 2\,x_2 \le 4$
$x_1, x_2 \ge 0$

We solve this problem using the proposed method in section 4.3, The correspondent linear program is given by:

$\text{Max. } Z_{RHF_{l2}} = 5x_1 + 4\,x_2$

subject to:

$\frac{2}{10}x_1 + \frac{3}{10}\,x_2 \le \frac{5}{10}$

$\frac{4}{10}x_1 + \frac{2}{10}\,x_2 \le \frac{4}{10}$

$7x_1 + 5.5\,x_2 \le 10$

$x_1, x_2 \ge 0$

Applied simplex algorithm to solve the problem, we get $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{2}$, $\text{Max. } Z_{RHF_{l2}} = 7.5$

Therefore, the optimal solution is $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{2}$, $\text{Max. } Z_{RHF_l} = \frac{51}{4} * 7.5 = 95.625$

Now, Solve the Example 1 by Efficient Ranking Function Methods for Fully Fuzzy Linear Fractional Programming problems via Life Problems (Mustafa and Sulaiman, 2022)

$$\text{Maximize Z} = \frac{(4.5,8.5,12) \otimes \tilde{x}_1 \oplus (3.5,5,9) \otimes \tilde{x}_2}{(2.5,5.5,9) \otimes \tilde{x}_1 \oplus (2,3.5,6.5) \otimes \tilde{x}_2},$$

subject to:

$$(2,5,8) \otimes \tilde{x}_1 \oplus (3,6,10) \otimes \tilde{x}_2 \le (5,12,18)$$
$$(4,6,7) \otimes \tilde{x}_1 \oplus (2,4,8) \otimes \tilde{x}_2 \le (4,10,14)$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

By using 4.1 First Ranking Function in (Mustafa and Sulaiman, 2022), we get the following LPP

$$\text{Maximize } Z = \frac{8.67x_1 + 6.11x_2}{6.04\,x_1 + 4.26x_2},$$

subject to:

$$5.36\,x_1 + 6.72\,x_2 \le 12.36$$
$$5.75\,x_1 + 5.06\,x_2 \le 9.88$$
$$x_1, x_2 \ge 0$$

After solving the above LFPP by Sharma approach (Sharma, 1980) we get the following results:
$x_1 = 0.33, x_2 = 1.57, \text{Max.} Z = 1.43$

Problem 2: (Production Planning) (Das et al., 2017)

The corporation produces two types of goods, A and B, with multi-objective profit. Product A has a profit of around \$2, \$3, and \$6 per unit, whereas product B has a profit of approximately \$1, \$4, and \$7 per unit. First, the cost for each unit of the above items is around \$1, \$2, and \$5 respectively, and approximately \$2, \$3, and \$6 correspondingly. secondly, the yields profits of around \$2, \$4, and \$9 per unit, and approximately \$1, \$6, and \$7 per unit, respectively. The cost for each unit of the above items is around \$3, \$5, or \$6, and approximately \$2, \$3, or \$4, respectively. The raw material required for producing product A and B is about (3,4,8) units per pound and (2,6,7) units per pound, correspondingly. The available supply of this raw material is limited to approximately (5,9,19) pounds. The labor required to produce one unit of product A is about 1, 3, or 6 hours, whereas for product B it is approximately 2, 4, or 9 hours per unit. However, the total amount of labor available every day is approximately 4, 6, or 17 hours. Compute the ideal quantity of items A and B to produce in order to maximize the overall profit.

Solution:

Let x_1 and x_2 to be the quantity of components of A and B respectively to be produced. Then the above problem can be expressed as:

$$\text{Maximize } Z = \frac{(2,3,6) \otimes \tilde{x}_1 \oplus (1,4,7) \otimes \tilde{x}_2}{(1,2,5) \otimes \tilde{x}_1 + (2,3,6) \otimes \tilde{x}_2}, \frac{(2,4,9) \otimes \tilde{x}_1 \oplus (1,6,7) \otimes \tilde{x}_2}{(3,5,6) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2}$$

subject to:

$$(3,4,8) \otimes \tilde{x}_1 \oplus (2,6,7) \otimes \tilde{x}_2 \le (5,9,19)$$
$$(1,3,6) \otimes \tilde{x}_1 \oplus (2,4,9) \otimes \tilde{x}_2 \le (4,6,17)$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

By step 1 point 1: let $w = (0.5,0.5)$, then find to FFMOLFP problem can be printed as:

$$\text{Maximize } Z = \frac{(1,1.5,3) \otimes \tilde{x}_1 \oplus (0.5,2,3.5) \otimes \tilde{x}_2}{(0.5,1,2.5) \otimes \tilde{x}_1 \oplus (1,1.5,3) \otimes \tilde{x}_2}, \frac{(1,2,4.5) \otimes \tilde{x}_1 \oplus (0.5,3,3.5) \otimes \tilde{x}_2}{(1.5,2.5,3) \otimes \tilde{x}_1 \oplus (1,1.5,2) \otimes \tilde{x}_2},$$

Therefore, we get FFLFP programming by using step 1 point 2 & 3

$$\text{Maximize } Z = \frac{(2,3,7.5) \otimes \tilde{x}_1 \oplus (1,5,7) \otimes \tilde{x}_2}{(2,3.5,5.5) \otimes \tilde{x}_1 \oplus (2,3,5) \otimes \tilde{x}_2},$$

subject to:

$$(3,4,8) \otimes \tilde{x}_1 \oplus (2,6,7) \otimes \tilde{x}_2 \le (5,9,19)$$
$$(1,3,6) \otimes \tilde{x}_1 \oplus (2,4,9) \otimes \tilde{x}_2 \le (4,6,17)$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

By using step 2 we convert the FFLFP problem to a revised harmonious fuzzy (RHF_l) problem as follows:

$$\text{Max. } Z_{RHF_l} = 2 * [(2x_1 + 1x_2) * (2x_1 + 2x_2)]/[(2x_1 + 1x_2) + (2x_1 + 2x_2)]$$

subject to:

$$3x_1 + 2\,x_2 \le 5$$
$$x_1 + 2\,x_2 \le 4$$
$$x_1, x_2 \ge 0$$

By using step 3 with section 4 (proposed method) and step 4.3 we get optimal solution as following:
After simplify, we have the follow:
Max. Z_{RHF_l} = Max. $Z_{RHF_{l1}}$ * Max. $Z_{RHF_{l2}}$
subject to:
$3x_1 + 2\,x_2 \leq 5$
$x_1 + 2\,x_2 \leq 4$
$x_1, x_2 \geq 0$
where
Max. $Z_{RHF_{l1}} = 4x_1 + 2\,x_2$
subject to:
$3x_1 + 2\,x_2 \leq 5$
$x_1 + 2\,x_2 \leq 4$
$x_1, x_2 \geq 0$
Using simplex method to solve the problem, we get $x_1 = \frac{5}{3}$, $x_2 = 0$, Max. $Z_{RHF_{l1}} = \frac{20}{3}$
Max. $Z_{RHF_{l2}} = 4x_1 + 4\,x_2/[(2x_1 + 1x_2) + (2x_1 + 2x_2)]$
subject to:
$3x_1 + 2\,x_2 \leq 5$
$x_1 + 2\,x_2 \leq 4$
$x_1, x_2 \geq 0$
We solve this problem using the proposed method in section 4, The equivalent linear program is given by:
Max. $Z_{RHF_{l2}} = 4x_1 + 4\,x_2$
subject to:
$\frac{9}{20}x_1 + \frac{6}{20}\,x_2 \leq \frac{15}{20}$
$\frac{3}{20}x_1 + \frac{6}{20}\,x_2 \leq \frac{12}{20}$
$12x_1 + 6\,x_2 \leq 20$
$x_1, x_2 \geq 0$
Using simplex algorithm to solve the problem, we get $x_1 = \frac{1}{2}$, $x_2 = \frac{7}{4}$, Max. $Z_{RHF_{l2}} = 9$.
Therefore, the optimal solution is $x_1 = \frac{1}{2}$, $x_2 = \frac{7}{4}$, Max. $Z_{RHF_l} = \frac{20}{3} * 9 = 60$.
Now, Solve the Example 2 by Efficient Ranking Function Methods for Fully Fuzzy Linear Fractional Programming problems via Life Problems (Mustafa and Sulaiman, 2022)

$$\text{Maximize Z} = \frac{(2,3,7.5) \otimes \tilde{x}_1 \oplus (1,5,7) \otimes \tilde{x}_2}{(2,3.5,5.5) \otimes \tilde{x}_1 \oplus (2,3,5) \otimes \tilde{x}_2},$$

subject to:

$$(3,4,8) \otimes \tilde{x}_1 \oplus (2,6,7) \otimes \tilde{x}_2 \leq (5,9,19)$$
$$(1,3,6) \otimes \tilde{x}_1 \oplus (2,4,9) \otimes \tilde{x}_2 \leq (4,6,17)$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

By using 4.1 First Ranking Function in (Mustafa and Sulaiman, 2022), we get the following LPP

$$\text{Maximize Z} = \frac{4.57\text{x}_1 + 4.76x_2}{3.86\text{ x}_1 + 3.47x_2},$$

subject to:

$$5.2\text{ x}_1 + 5.283\text{ x}_2 \leq 11.94$$
$$3.7\,x_1 + 5.51\text{ x}_2 \leq 10.17$$
$$x_1, \text{x}_2 \geq 0$$

After solving the above LFPP by Sharma approach (Sharma, 1980), we get the following results:

$x_1 = 1.32, x_2 = 0.96,$ Max. $Z = 1.25$

10. Simulate the Numerical Results

Here, simulates the numerical results of proposed approach with efficient ranking function approach (Mustafa and Sulaiman, 2022) of examples 1 and 2 respectively,

Table 1: Comparison between proposed approach and efficient ranking function approach

Approach	Solution of Example 1	Solution of Example 2
Revised Harmonious Fuzzy Technique	$x_1 = \frac{1}{4}, x_2 = \frac{3}{2},$ Max. $Z_{RHF_l} = 95.625$	$x_1 = \frac{1}{2}, x_2 = \frac{7}{4},$ Max. $Z_{RHF_l} = 60.$
Efficient Ranking Function Approach	$x_1 = 0.33, x_2 = .57,$ Max. $Z = 1.43$	$x_1 = 1.32, x_2 = 0.96,$ Max. $Z = 1.25$

The table demonstrates that the revised harmonious fuzzy technique yields promising results. In contrast, the efficient ranking function approach, as exemplified by $Maz\ Z$ in both examples (1,2), exhibits weaknesses, these results highlight the strength and efficiency of the suggested approach and algorithm.

11. Results and Discussion

The effectiveness of the Revised Harmonious Fuzzy Technique (RHFT) is showcased in addressing Fuzzy Multi Linear Fractional Programming (FFMOLFP) challenges, as illustrated through the numerical instances shared in this research.

11.1 Main Discoveries

Efficiency; RHFT effectively converts FFMOLFP issues into Linear Programming (LP) problems reducing computational complexity and aiding in solution identification.

Precision; The technique precisely accommodates the uncertainties and ambiguities, in FFMOLFP problems by using numbers to represent parameters and variables. This method ensures dependable solutions that mirror the nature of real-world situations.

Adaptability; RHFT can be applied to a variety of FFMOLFP problems as evidenced by the applications in production planning scenarios outlined. The methods flexibility allows for an approach, to tackling optimization challenges.

11.2 Discussion

The RHFT method described in this article offers an advancement, over approaches for addressing FFMOLFP issues. It distinguishes itself by incorporating Fuzzy Harmonic Mean (FHM) which effectively combines functions into a unified representation capturing the essence of conflicting objectives within an uncertain context. Furthermore, the proposed method systematically transforms FFMOLFP challenges into LP problems providing a clear and structured approach to solving intricate optimization dilemmas. Additionally, its capability to handle parameters and constraints makes RHFT particularly beneficial, for addressing practical optimization challenges where decision variables and objectives often involve uncertainties.

12. Conclusion

This paper has presented a novel approach for solving FFMOLFP problems, employing the Revised Harmonious Fuzzy Technique (RHFT). This method utilizes the strength of fuzzy set theory and harmonious search algorithms to manage the uncertainties and ambiguities, in real-world decision-making situations. The proposed approach systematically converts FFMOLFP issues into a sequence of linear programming problems enabling the identification of optimal solutions. The crucial steps include transforming the FFMOLFP problem into a FFLFP dilemma using a weighting strategy, followed by further conversion into an RHFT problem by introducing the fuzzy harmonic mean (FHM). Eventually the RHFT quandary is turned into an LP predicament, which is resolved using the simplex method to derive the solution. The efficiency of RHFT has been showcased through two examples. These instances exhibit how this method can address production planning challenges with aims and uncertain variables. RHFT offers a computationally efficient approach to managing FFMOLFP problems providing valuable insights, for decision makers across diverse domains.

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