#### ISSN (print ):2218-0230, ISSN (online): 2412-3986, DOI: http://dx.doi.org/10.21271/zjpas

# **RESEARCH PAPER**

## Determination the Location of an Air Inception Point for Different Configurations of Stepped Spillways using CFD

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### ABSTRACT:

Stepped spillways are a common way for embankment dams to let water out when flood happens. The goal of this research is to assess the impact of various factors on the position of the inception point for a stepped spillway, such as the number of steps, step angle, and flow rate. For this purpose a CFD-FLUENT code, based on Finite Volume Method combined with Volume of Fluid to capture free-surface flow properties is used to model the hydraulics of skimming flow through uniform stepped spillways with moderate slopes typical of embankment dams, and the realizable k- $\varepsilon$  model is used to determine the turbulence closure. The code was tested against published experimental data of (Hunt and Kadavy 2009) for this purpose. Numerical results of ANSYS-fluent compared to the experimental data, and it was found to be in agreement. Then, the impact of the aforementioned factors on the location of an air inception point and its length were next investigated by simulating 45 cases in total with constant chute slope 26°, three varied step angles (0°, 6°, 12°), three different step numbers(20, 30, 40), and five different skimming flows (0.16, 0.32, 0.48, 0.64, and 0.8)  $\frac{m^2}{s}$ .

KEY WORDS: Fluent, VOF, Inception point, Stepped spillway, Realizable  $k - \varepsilon$  Model, CFD. DOI: <u>http://dx.doi.org/10.21271/ZJPAS.34.6.1</u> ZJPAS (2022), 34(6);1-7 .

### 1. INTRODUCTION

A large amount of kinetic energy may be dissipated owing to the steps in the spillways. Because of this, a smaller stilling basing is needed at the spillway toe. Dissipaters of energy for spillway may be found in stepped spillways (Zhang *et al.* 2012). The round stepped spillway model's inception point is closer to the spillway crest than the flat stepped spillway. (Wan, Raza and Chen 2019).

Zaid P. O E-mail: pshtewan.zaid@gmail.com Article History: Received: 20/04/2022 Accepted: 26/07/2022 Published: 20/12 /2022 The efficiency of a stepped spillway is influenced by the amount of air that is entrained, due to the reason that cavitation often occurs in the nonaerated flow zone, accurate data of air entrainment are important. The distance from the inception point to the non-aerated flow zone is indicated by the location. Because the inception point can be gets closer, the aerated and non-aerated flow accurately anticipated zones may be by determining the exact position (Amador, santchez and Dolz 2006). Total air concentration refers to the amount of air that has been entrained and trapped in a stepped spillway. To avoid cavitation damage, a concentration of 5-8 percent pseudobottom air is necessary (y, Yasuda and Takahashi 2004).

A rapid increase in air concentration occurs at the pseudo-bottom and soon approaches the critical

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2

point, after which it rises to the maximum value. Low pressure may cause cavitation damage, which can be prevented with air entrainment (Hunt and Kadavy 2009). When the boundary layer thickness reaches 80% of the flow depth, the stepped spillway's inception point is reached. Upstream of the inception point, the flow is smooth, whereas downstream, the flow is quite turbulent (Chanson, H 1996).

#### 2. METHODOLOGY

Computational fluid dynamics is a subfield of fluid mechanics in which numerical analysis is used in association with a computer to solve issues involving fluid flow, heat and mass transfer, and chemical interactions. Since its creation, engineers have used it extensively in a broad variety of application (Abo et al 2013). The Navier-Stokes equation, which is based on the conservation of mass, energy, and momentum, may be solved numerically with the use of a computer. Computer-aided design (CFD) software is used to model stepped spillways. The Navier-Stokes equation is discretized using fluent software by the finite volume technique (FVM). Dynamic equations in CFD are most typically discretized using an FVM approach. This approach uses algebraic equations instead of numerical solutions to solve the governing formula. The control volume's integration equations are solved implicitly (Raza, Wan and Mehmood 2021).

$$\frac{\partial v}{\partial \chi} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Where: u= Fluid velocity in x direction in (m/s), v= Fluid velocity in y direction in (m/s)

#### X-Momentum equation:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\partial uv)}{\partial y} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
(2)

Y-Momentum equation:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\partial v u)}{\partial x} + \frac{\partial(\partial v^2)}{\partial y} = \rho g_y - \frac{\partial \mathcal{P}}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$
(3)

Where:

 $\rho$ =fluid density (kg/m3),  $\mu$ =fluid viscosity (Pa.s), gx=gravity acceleration in the x-direction (m/s2), gy=gravity acceleration in the y-direction (m/s2),  $\partial p/\partial x$  =pressure gradient in x-direction (Pa/m),  $\partial p/\partial y$ =pressure gradient in y-direction (Pa/m)

The steps in a spillway have a considerable impact on the roughness. Since roughness causes flow to be very turbulent, this is the case with a stepped spillway. Bouncing two-phase water over an inclined spillway may produce even more turbulent flow. Consequently, turbulence is predicted using a turbulent model. For high Reynolds number turbulent flow, the realizable  $k - \varepsilon$  model was proposed, which is based on the realizable constraints. Due of the huge turbulent Reynolds number, a unique equation for the dissipation rate  $(\varepsilon)$  is suggested, which yields better results than prior models. The realizable k- $\epsilon$ model's performance has been enhanced and the recirculation flow results are satisfactory. Equations (4) and equation (5), are respectively, for the transmission of turbulence kinetic energy (k) and turbulence dissipation rate ( $\epsilon$ ) (Raza, Wan and Mehmood 2021).

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial xi}(\rho k u j) = \frac{\partial}{\partial xi} \left[ \left( \mu + \frac{\mu t}{\sigma k} \right) \frac{\partial k}{\partial xj} \right] + G_k - \rho \epsilon \quad (4)$$

$$\frac{\partial}{\partial t}(\rho\epsilon) + \frac{\partial}{\partial xj}(\rho\epsilon uj) = \frac{\partial}{\partial xj} \left[ \left( \mu + \frac{\mu t}{\sigma\epsilon} \right) \frac{\partial\epsilon}{\partial xj} \right] + C_{\epsilon 1} \frac{\epsilon}{k} G_k - C_{\epsilon 2} \rho \frac{\epsilon^2}{k}$$
(5)

**VOF** (Volume of Fluid): Hirt and Nichols created the VOF multiphase model. It is most often utilized when a process contains more than one step. This approach is based on the premise that the various stages will remain separate. Tracking air-water contact in free-surface flow is the goal of this study's research. Because of this, VOF is a useful tool for determining the location of the interface between distinct phases. Other multiphase models simply track bubbles, but the VOF technique records the interface as a mixed cell. The VOF method when the interface is taken into account. In VOF models, the volume percentage of fluid (VOF) may be specified as 0 if the cell does not contain water and 1 if it is totally filled with water. The air and water volume fractions are represented by  $\alpha_a$  and  $\alpha_w$ , respectively (Raza, Wan and Mehmood, 2021).  $\alpha_a + \alpha_w = 1$ (6)

ZANCO Journal of Pure and Applied Sciences 2022

When it comes to stepped spillway simulations, CFD is frequently used because of recent advancements software and in hardware technologies. Authors utilize ANSYS, fluid flow software, to study various flow characteristics. Researchers use numerical data from their experiments to check the amount of their findings and to gain confidence in the accuracy of their predictions. It is necessary to pre-define the geometry, mesh, and processing steps in this software. These procedures are described in the following section. To be more confident we try to validate one model for two different flows experimentally tested by (Hunt and Kadavy 2009).

**Model geometry, results, and discussion:** Flood flow inception points were evaluated using a twodimensional model of a stepped spillway. Using the flume's whole length, the model was built with 2.4m tall and the spillway model has a vertical drop of 1.5 meters. The model's chute has a 4H: 1V slope and was built with a broad-crested weir. The model has step heights of 38 mm. It is at this point that the weir's downstream border is aligned with station 0.0 m and step 0. At step 40, the spillway's downstream toe measures 6.1 meters (Chakib 2013). 2D model is shown in the figure 1.



**Figure 1:** Geometry of a stepped spillway model (Hunt and Kadavy 2009).

The numerical CFD code supports inlet, outlet, opening (free surface), symmetry, and wall boundary conditions. Hexahedral meshes with grid side size (0.005m) used.



Figure 2: Typical mesh and inflation technique.



Figure 3: Inception point at step 9 for (q=0.11m2/s)



Figure 4: Inception point at step 22 for (q=0.28m2/s)

| Table 1: Inception | on point location | (Observed & I | Numerical) |
|--------------------|-------------------|---------------|------------|
|                    |                   | (             |            |

| q (m2/s) | Observed | Numerical |
|----------|----------|-----------|
| 0.11     | step 9   | step 9    |
| 0.28     | step 22  | step 22   |

Fluent was used to model the flow through a flatsloped stepped spillway. Turbulence flow and the free surface were modeled using the VOF model and the realizable k- $\varepsilon$  model, respectively. The numerical and experimental findings show a high degree of agreement. There was evidence to suggest that as the flow increased, the inception point moved closer to the basin bottom.

#### 4. RESULTS AND DISCUSSION

As part of this study, one moderate chute slope with three different step slopes, three different step numbers, and five various skimming flow are developed and evaluated.

**Numerical Model:** The influence of step height, step angle, and different flows on the characteristics of skimming flow conditions is studied in three different configurations shown below with constant chute slope  $26^{\circ}$ , 45 cases in total. Configuration A is consist of 20 steps of

ZANCO Journal of Pure and Applied Sciences 2022

11.9cm height with three different step angles  $(0^{\circ}, 6^{\circ}, 12^{\circ})$ , configuration B is consist of 30 steps of 8.1cm height with three different step angles  $(0^{\circ}, 6^{\circ}, 12^{\circ})$ , and configuration C is consist of 40 steps of 6.1cm height with three different step angles  $(0^{\circ}, 6^{\circ}, 12^{\circ})$ . All configurations are tested for five different skimming flows (0.16, 0.32, 0.48, 0.64, and 0.8)  $m^2/s$ .

4



Figure 5: Typical model of stepped spillway configurations

Table 2: Detail of stepped spillway configurations

| ston | S h(  | SI           | sten     | chute     | $a(\mathbf{m}^2/\mathbf{s})$      |
|------|-------|--------------|----------|-----------|-----------------------------------|
| step | 5.11( | <b>5.</b> L  | step     | chute     | <b>q</b> ( <b>m2</b> / <b>8</b> ) |
|      | m)    | ( <b>m</b> ) | angle(α) | angle (0) |                                   |
| 20   | 0.1   | 0.2          | 0        | 26        | 0.16                              |
| 20   | 0.1   | 0.2          | 0        | 26        | 0.32                              |
| 20   | 0.1   | 0.2          | 0        | 26        | 0.48                              |
| 20   | 0.1   | 0.2          | 0        | 26        | 0.64                              |
| 20   | 0.1   | 0.2          | 0        | 26        | 0.8                               |
| 20   | 0.1   | 0.2          | 6        | 26        | 0.16                              |
| 20   | 0.1   | 0.2          | 6        | 26        | 0.32                              |
| 20   | 0.1   | 0.2          | 6        | 26        | 0.48                              |
| 20   | 0.1   | 0.2          | 6        | 26        | 0.64                              |
| 20   | 0.1   | 0.2          | 6        | 26        | 0.8                               |
| 20   | 0.1   | 0.2          | 12       | 26        | 0.16                              |
| 20   | 0.1   | 0.2          | 12       | 26        | 0.32                              |
| 20   | 0.1   | 0.2          | 12       | 26        | 0.48                              |
| 20   | 0.1   | 0.2          | 12       | 26        | 0.64                              |
| 20   | 0.1   | 0.2          | 12       | 26        | 0.8                               |
| 30   | 0.0   | 0.1          | 0        | 26        | 0.16                              |
| 30   | 0.0   | 0.1          | 0        | 26        | 0.32                              |
| 30   | 0.0   | 0.1          | 0        | 26        | 0.48                              |
| 30   | 0.0   | 0.1          | 0        | 26        | 0.64                              |
| 30   | 0.0   | 0.1          | 0        | 26        | 0.8                               |
| 30   | 0.0   | 0.1          | 6        | 26        | 0.16                              |
| 30   | 0.0   | 0.1          | 6        | 26        | 0.32                              |
| 30   | 0.0   | 0.1          | 6        | 26        | 0.48                              |
| 30   | 0.0   | 0.1          | 6        | 26        | 0.64                              |
| 30   | 0.0   | 0.1          | 6        | 26        | 0.8                               |
| 30   | 0.0   | 0.1          | 12       | 26        | 0.16                              |
| 30   | 0.0   | 0.1          | 12       | 26        | 0.32                              |
| 30   | 0.0   | 0.1          | 12       | 26        | 0.48                              |
| 30   | 0.0   | 0.1          | 12       | 26        | 0.64                              |
| 30   | 0.0   | 0.1          | 12       | 26        | 0.8                               |
| 40   | 0.0   | 0.1          | 0        | 26        | 0.16                              |

| 26 | 0.8  | Configuration | nodes |
|----|------|---------------|-------|
| 26 | 0.16 |               |       |
| 26 | 0.32 | A (20steps)   | 93400 |
| 26 | 0.48 | B (30steps)   | 93457 |
| 26 | 0.64 | E (30steps)   | 95091 |
|    | 0.0  | C (40steps)   | 03901 |

| Configuration | skewness | Aspect<br>ratio | Orthogonal quality |
|---------------|----------|-----------------|--------------------|
| A (20steps)   | 0.106    | 1.226           | 0.979              |
| B (30steps)   | 0.102    | 1.222           | 0.98               |
| C (40steps)   | 0.061    | 1.366           | 0.979              |

No.

elements

92159

92299

84715

Inflation

growth

rate

1.2

1.2

1.2

No.

inflation

layer

4

4

4

**Boundary conditions:** Numerical modeling relies on assigning correct boundary conditions to properly describe flow behavior while using as few computer resources as possible. The

| 40 | 0.0 | 0.1 | 0  | 26 | 0.32 |
|----|-----|-----|----|----|------|
| 40 | 0.0 | 0.1 | 0  | 26 | 0.48 |
| 40 | 0.0 | 0.1 | 0  | 26 | 0.64 |
| 40 | 0.0 | 0.1 | 0  | 26 | 0.8  |
| 40 | 0.0 | 0.1 | 6  | 26 | 0.16 |
| 40 | 0.0 | 0.1 | 6  | 26 | 0.32 |
| 40 | 0.0 | 0.1 | 6  | 26 | 0.48 |
| 40 | 0.0 | 0.1 | 6  | 26 | 0.64 |
| 40 | 0.0 | 0.1 | 6  | 26 | 0.8  |
| 40 | 0.0 | 0.1 | 12 | 26 | 0.16 |
| 40 | 0.0 | 0.1 | 12 | 26 | 0.32 |
| 40 | 0.0 | 0.1 | 12 | 26 | 0.48 |
| 40 | 0.0 | 0.1 | 12 | 26 | 0.64 |
| 40 | 0.0 | 0.1 | 12 | 26 | 0.8  |

**Geometry creation:** ANSYS workbench or any other CAD programs may be used to construct the spillway geometry, which is subsequently exported to ANSYS-FLUENT. To develop the geometry for this investigation, the shape of 2D stepped spillway created by AutoCAD program then saved as SAT file and imported to ANSYS.

**Mesh Generations:** More accurate results may be achieved with finer meshes; this is due to a considerable influence on the correctness of the results. Various mesh refinement methods, such as inflations on walls, the bed, and the steps surrounding, were applied in these experiments using hexahedral meshes with grid side sizes (0.005m). CFD- Meshing size approaches were used to modify the flow pattern around the steps as well, in order to capture the tiny scales caused by flow separations. The detail of meshing for all configurations Shown in Table 3:

Table 3: Mesh detail and quality

Configuration

No.

| ZANCO Journal of Pure and Applied Sciences 2022 |
|---|
|---|

numerical ANSYS-FLUENT code supports inlet, outlet, opening (free surface), symmetry, and wall boundary conditions.

Inlet: Stepped spillway models have a velocity input at their inlet. The velocity of the stepped spillway changes depending on the flow rate.

Outlet: Stepped spillways have pressure outlets with no backwater flow. Outlet backwater flow is zero.

Wall: The walls of the stepped spillway are regarded as static, nonslip surfaces. Therefore, they are considered as stationary walls.

SIMPLE is used in numerical modeling to relate pressure and motion. The SIMPLE approach is utilized for transient simulation since simulation is transient in this scenario; hence we employ the SIMPLE process for velocity-pressure coupling. For momentum, turbulent kinetic energy dissipation rate, and pressure delayed option (PRESTO), second order upwind and first order upwind techniques are used. As a final step, the numerical model outputs, comprises the position of the inception point as follows:



**Figure 6:** graphical result of stepped spillway inception point (20 steps,  $\alpha=0$ , q=0.32m2/s)

Table 4: Results of all spillway configurations with

different unit discharges

| ste<br>ps | step<br>angl | chute<br>angle | q(m2<br>/s) | inception(<br>step) | inception(le<br>ngth) |
|-----------|--------------|----------------|-------------|---------------------|-----------------------|
| 20        | 0            | 26             | 0.16        | 1                   | 0.277                 |
| 20        | 0            | 26             | 0.32        | 3                   | 0.82                  |
| 20        | 0            | 26             | 0.48        | 5                   | 1.384                 |
| 20        | 0            | 26             | 0.64        | 8                   | 2.059                 |
| 20        | 0            | 26             | 0.8         | 10                  | 2.745                 |
| 20        | 6            | 26             | 0.16        | 1                   | 0.277                 |
| 20        | 6            | 26             | 0.32        | 3                   | 0.777                 |
| 20        | 6            | 26             | 0.48        | 5                   | 1.384                 |
| 20        | 6            | 26             | 0.64        | 7                   | 2.114                 |
| 20        | 6            | 26             | 0.8         | 10                  | 2.769                 |
| 20        | 12           | 26             | 0.16        | 1                   | 0.277                 |

| 20 | 12 | 26 | 0.32 | 3     | 0.831 |
|----|----|----|------|-------|-------|
| 20 | 12 | 26 | 0.48 | 6     | 1.567 |
| 20 | 12 | 26 | 0.64 | 18    | 4.954 |
| 20 | 12 | 26 | 0.8  | basin | basin |
| 30 | 0  | 26 | 0.16 | 2     | 0.266 |
| 30 | 0  | 26 | 0.32 | 4     | 0.741 |
| 30 | 0  | 26 | 0.48 | 8     | 1.392 |
| 30 | 0  | 26 | 0.64 | 11    | 2.037 |
| 30 | 0  | 26 | 0.8  | 15    | 2.777 |
| 30 | 6  | 26 | 0.16 | 2     | 0.266 |
| 30 | 6  | 26 | 0.32 | 4     | 0.741 |
| 30 | 6  | 26 | 0.48 | 8     | 1.396 |
| 30 | 6  | 26 | 0.64 | 22    | 4.073 |
| 30 | 6  | 26 | 0.8  | 30    | 5.555 |
| 30 | 12 | 26 | 0.16 | 2     | 0.371 |
| 30 | 12 | 26 | 0.32 | 5     | 0.928 |
| 30 | 12 | 26 | 0.48 | 9     | 1.579 |
| 30 | 12 | 26 | 0.64 | 25    | 4.581 |
| 30 | 12 | 26 | 0.8  | 30    | 5.579 |
| 40 | 0  | 26 | 0.16 | 2     | 0.278 |
| 40 | 0  | 26 | 0.32 | 5     | 0.695 |
| 40 | 0  | 26 | 0.48 | 10    | 1.349 |
| 40 | 0  | 26 | 0.64 | 22    | 3.06  |
| 40 | 0  | 26 | 0.8  | 37    | 5.132 |
| 40 | 6  | 26 | 0.16 | 2     | 0.278 |
| 40 | 6  | 26 | 0.32 | 6     | 0.751 |
| 40 | 6  | 26 | 0.48 | 10    | 1.365 |
| 40 | 6  | 26 | 0.64 | 23    | 3.199 |
| 40 | 6  | 26 | 0.8  | 37    | 5.147 |
| 40 | 12 | 26 | 0.16 | 2     | 0.296 |
| 40 | 12 | 26 | 0.32 | 5     | 0.777 |
| 40 | 12 | 26 | 0.48 | 10    | 1.391 |
| 40 | 12 | 26 | 0.64 | 29    | 4.033 |
| 40 | 12 | 26 | 0.8  | 39    | 5.424 |



**Figure 7:** Inception point with respect to step height (Flat steps)

ZANCO Journal of Pure and Applied Sciences 2022



6

Figure 8: Inception point with respect to step height (Sloped steps  $\alpha=6^{\circ}$ )



**Figure 9:** Inception point with respect to step height (Sloped steps  $\alpha=12^{\circ}$ )

From Figures 7, Figure 8, and Figure 9, it is clear the slope of steps has more effect on larger flows and the inception point move downstream for less step heights as the slope increased, except some cases which the increase step slop has no effect of increasing inception length, this cases occur for large flows and large step slopes at the same time.



**Figure 10:** Inception point with respect to step slope (20 steps)



**Figure 11:** Inception point with respect to step slope (30 steps)



**Figure 12:** Inception point with respect to step slope (40 steps)

Flat and slopped stepped spillway have the same inception point length for the same discharge but the difference starts when the discharge become large amount and the difference appear more with increasing flow. From Figure 10, Figure 11, and Figure 12, it is clear that for the same step slope and discharge if the number of steps increased or the height of steps decreased, the length of inception will increase.

#### 5. CONCLUSION

The turbulent velocity operating at the air-water interface causes air to be entrained at the free surface of high-velocity flows in stepped channels. When the outside edge of the developing boundary layer reaches the free surface, air entrainment begins in the skimming flow regime and upstream of the inception point. Based on this research's computational outcomes, we may conclude:

• The inception point is nearer to the spillway crest with a bigger step height under the same discharge especially for large flow rates and

flat steps, but the results are in opposite direction when the steps are slopped and the effect is increased with increasing the step slope.

• All stepped spillway configurations have results that show the length of the inception point gets longer with the larger flow rate.

• Increasing step slope has more effect to increase length of inception point than decreasing step height, except in case of large flows and large step slopes at the same time, increasing  $\alpha$  will not affect the inception length.

• When comparing the flat stepped spillway model to the sloping stepped spillway model under the same flow, the position of the inception point in the flat stepped spillway model is closer to the spillway crest.

• Flat and slopped stepped spillway have the same inception point length for the same flow but the difference starts when the discharge become larger and the difference appear more with increasing flow.

• For the same step slope and discharge if the number of steps increased or the height of steps decreased, the length of inception will increase.

#### LIST OF ABBREVIATION

- $C_{\varepsilon 2}$  1.92 (model constant)
- $C_{\varepsilon 1}$  1.44 (model constant)
- $C_u$  coefficient of uniformity
- $G_u$  generation of k due to buoyancy
- $G_k$  generation of k due to fluid Shear
- *g* gravitational acceleration (m/s2)
- k turbulent kinetic energy (m2/s2)
- ks roughness height, ks =  $hcos\theta$  (m)
- $\alpha_a$  volume fraction of air (%)

- $\alpha_w$  volume fraction of water (%)
- ε turbulent dissipation rate
- μ molecular dynamic viscosity
- μt turbulent dynamic viscosity
- $\sigma_k$  1.0 (model constant)
- $\sigma_{\varepsilon}$  1.2(model constant)

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