ZANCO Journal of Pure and Applied Sciences

TYPE Original article
DOI http://dx.doi.org/10.21271/ZJPAS.37.1.2

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RECEIVED :26/09/2024
ACCEPTED :30/11/2024
PUBLISHED :28/02/2025

KEYWORDS:
Health,
Noise pollution,
Hearing loss,
Mathematical model,
Analysis.

Mathematical Model of Hearing Loss caused by Noise Hazard

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ABSTRACT

Hearing loss is a growing public health concern with serious implications for individual's quality of life. One major cause of hearing loss is exposure to loud noise. This study proposes a mathematical model of hearing loss caused by noise exposure using ordinary differential equations with the aim of providing a framework for understanding the dynamics of the impact of noise hazard. The model is analyzed using local and global stability to determine conditions under which the level of noise pollution remains constant or changes over time and the threshold level beyond which the effect of noise becomes uncontrollable. Sensitivity analysis is performed to determine parameters with greater influence and the ones to control in order to reduce impact of noise exposure. The findings highlight the importance of various parameters in the dynamics of hearing loss by noise exposure and in developing effective strategies for preventing and managing hearing loss.

1.Introduction

Hearing is one of the crucial senses and like vision is vital for distant warning and communication. It is used for creating alert, to communicate pleasure and also fear. Hearing enables one to identify and distinguish between objects within an environment based on the sound they create (Alberti, 2001). Sounds are forms of energy produced by the vibration of objects. Sound can travel through different types of matter such as solids, liquids and gases. It travels approximately 340 meters per second through air; however, it goes quicker through liquids or solids (Van Hemel, S.B et al., 2004), (Lee and Fleming, 2002), (Natarajan et al., 2023). The decibel (dB), which is the logarithm of the ratio of two different sound intensities or pressures, is the standard unit of measurement for sound level while Hertz (Hz) defined as cycles per second is the unit for frequency (National Research Council Committee on Disability Determination for Individuals with Hearing, 2004),(Natarajan, 2023). When sound interferes with regular activities such as sleep or conversation, it becomes noise.

The word noise comes from the Latin word “nausea", which means “sea sickness" (Kumar et al., 2004). It is a highly subjective term and has been defined by many psychologists. According to Kiely (1997), it is described as undesired sound, which may be thought of as the incorrect sound at the wrong place at the wrong time. In addition, Singal (2005) defines noise as any sound that is unwanted by the recipient and has the potential to harm the health and well-being of people. Noise is also defined as an undesired sound, a possible health danger, and a message that is released into the environment with the intention of having a negative impact on those who do not wish to hear it (Vanadeep and Krishnaiah, 2011). Noise is described in medical literature as a very loud sound capable of causing harm to the inner ear (Šušković and Fajt, 2012). Webster defines noise as “a sound that lacks agreeable musical quality or is noticeably loud, harsh or discordant”. It has global effects like air, water and other environmental pollutant, it is now recognized as a serious environmental issue.(Agarwal, 2005)

According to Singh (1991) noise pollution is, “the state of discomfort and restlessness caused to humans by unwanted high intensity sound", meaning, unwanted sound released into the atmosphere leading to health hazards. Noise could originate from a variety of sources. There are three categorized primary sources of noise, which include: natural, biological and artificial sources (Mahandiyan, 2006),(Schmidt, 2005) . Aside the several negative impacts of noise on human health, its effects are generally classified under four different types: psychological, physiological, reduction productivity and physical effects.

Psychological effects: Humans and animals behaviour have been changed because of high level of noise. It is observed that unwanted noise frequently causes aggravation, frustration, and weariness, resulting in poor performance, efficiency, and a high rate of mistakes. Noise has negative impacts on efficiency, especially in youngsters. When houses or schools are located near sources of noise such as roads or airports, cognitive development decreases.

Physiological effects: Noise pollution may induce annoyance, aggravation, anxiety, strain, and stress, which can lead to changes in blood hormone levels, these could also cause a change in human bodies. Record has it that the noise level of 55 dBA is sufficient to cause serious annoyance in outdoor environment. Noise pollution of various sorts caused by varying level of noise may cause high blood pressure, heart diseases, dilation of pupils of the eyes, tensing of the voluntary and involuntary muscles, diminution of gastric secretion, neuromuscular tension, nervousness, stomach and intestinal diseases such as ulcer etc. Lung damage occurs at 190 dBA. A very high-level noise caused by sonic booms or explosion may lead to termination of pregnancy in early stages (Humes, L.E., 2019).

Reduction productivity effects: Noise has a variety of impacts on humans, including speaking interference, irritation, sleep disruption, and other issues (Singh, 1991). Speech interference simply refers to a person's inability to communicate and hear a sound because of background noise. The most crucial and also the most difficult use of the auditory system is speech reception. Noise can either mask speech, rendering it inaudible, or make just certain frequencies audible but with diminished intelligibility . Noise decreases the depth and quality of sleep, which has a negative impact on one's mental and physical health. Furthermore, noise levels of more than 40 decibels at night have been linked to sleep disruption (Humes, L.E., 2019). Low-frequency noise, even as low as 50 to 60 dBA, can influence the brain's higher centers, disrupting sleep patterns and preventing deep sleep (Agarwal, 2005). In addition, it has been proven in both laboratory and field research that employees who are exposed to occupational noise have poor performance on cognitive tasks. Although, the effects vary depending on the kind of noise and the activity at work (Qutubuddin et al., 2012).

Physical effects: Damage to the human hearing system caused by various forms of noise is referred to as auditory effects. The human ear is a very sensitive organ. Hearing will be affected if the hearing systems are harmed in any manner, whether by high noise levels or illnesses which affects the brain, auditory nerve, or auditory ossicles (González, 2014). The impacts of high-intensity noise on humans are indicated by a hearing threshold of 0 dBA, a pain threshold of 120 dBA, ear pain threshold of 140 dBA, and ear drum damage of 160 dBA (Vasudevan, 2006). In the 90 dBA range, auditory fatigue can occur, along with side effects such as whistling and buzzing in the ears. Continuous noise exposure might result in deafness. At 100 dBA, permanent hearing loss occurs (Singh, 1991). If hearing loss goes unnoticed, it can have a negative impact on a person's ability to operate. In general, hearing loss is caused by several factors such as noise level and the daily noise exposure period, heredity (genetics), birth complications, aging (presbycusis), the sound wave pressure and frequency, certain viral diseases, chronic ear infections (Van Hemel, S.B et al., 2004), (Kouilily et al., 2018b), (Kouilily et al., 2018a).

It has been established that once exposed to loud noise, temporary hearing loss can occur which progresses with continuous exposure to loud noise or sound (Van Hemel, S.B et al., 2004),(Adisesh et al. 2022). When it becomes permanent as a result of continuous exposure to extremely loud noise it is referred to as permanent noise-induced hearing loss and cannot be regain (Liverman et al. 2016), (Van Hemel, S.B et al., 2004), (Adisesh et al. 2022), (Sataloff, R.T, 2006). According to Shapiro (2019), there are numerous negative impacts of hearing loss on the individual which include falls that can result to serious injuries or even death. He further assert that even a "mild degree of hearing loss triples the risk of an accidental fall" (Shapiro, 2019). The Center for Disease Control and Prevention (CDC) also affirm that 25% of the elderly people falls each year and the outcome of these falls were often emergencies cases, broken bones, traumatic brain injuries, hospitalizations or death (Chen et al., 2020).

Researchers assert that if a temporary hearing loss is as a result of the environment (for instance, loud noise at a factory or other sources), it resolves after few minutes, days or weeks of treatment and/or cautionary measures depending on the level of exposure. They also states that precaution among other measures are required to protect the ears against hearing loss (Dickson, 1953), (Van Hemel, S.B et al., 2004), (Adisesh et al. 2022), (Sataloff, R.T, 2006). In addition, Clason (2021) affirm that most cases of hearing loss is "mild and will go away quickly" (Liverman et al. 2016). Consequently, individuals with temporary hearing loss could have such cases resolved within a short time duration and eventually be considered still susceptible to loud noise when re-exposed to any noisy environment.

According to the World Health Organization (WHO), more than 430 million people, or 5% of the world's population, require rehabilitation to address their “disabling" hearing loss and 34 million of these are children. By 2050, it is estimated that this number will increase to more than 700 million individuals or one out of every 10 people (Humes, L.E., 2019). Mathematical models have been formulated to analysed the function and dysfunction of the inner ear by using partial differential equation while other researchers used SIR model to describe the dynamics of hearing loss caused by viral Infection such as mumps (contagion factor) and social factors (Kouilily et al., 2018b), (Kouilily et al., 2018a).

In this paper, we propose $SEHRE$ model to study the dynamics of hearing loss caused by hazardous exposure to noise. This is done by considering the model in (Kouilily et al., 2018b), (Kouilily et al., 2018a)] and introducing the exposed class with the aim of accounting for the susceptible individuals losing their hearing abilities due to excessive exposure to noise hazard. Consequently, the model which is derived using ordinary differential equations, evaluates the impact of noise hazard on human health with focus on hearing loss. The paper has five sections, in Section 2, we present the model description and derive the basic reproduction number. Model analysis consisting of the stability analysis of noise-free, noise-endemic steady states, and the sensitivity analysis of parameters, is discussed in Section 3. We use numerical simulation to also show the dynamical behaviour of our results in Section 4. Finally, the discussion of results is captured in section 5.

2.Mathematical Model

2.1 Model description

In this section, we introduce a new mathematical model $SEHRE$ by using ordinary differential equations to study the dynamics of hearing loss caused by exposure to noise. The entire population is divided into four compartments: the susceptible individuals (those with normal hearing but are likely to loss same when exposed), S, the individuals exposed to noise hazard, E, the affected individuals (those suffering loss of hearing), H and lastly, the removed individuals at time t (those having temporary hearing loss with the possibility of regaining or resolving the issue within few weeks of treatment and/or cautionary measures.), R. The interaction between the four compartments is capture in Figure 1.

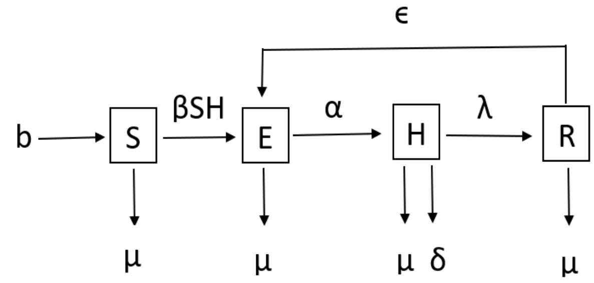


Figure 1: Model diagram for the dynamics of hearing loss

The derived system of equations of the model is given in Eq. (1) while the description of the parameters used is given in Table 1.

$$\frac{dS}{dt} = b - \beta SH - \mu S$$
$$\frac{dE}{dt} = \beta SH + \epsilon R - (\alpha + \mu)E$$
$$\frac{dH}{dt} = \alpha E - (\lambda + \mu + \delta)H$$
$$\frac{dR}{dt} = \lambda H - (\mu + \epsilon)R$$

with the initial condition: $S(0) > 0, E(0) \geq 0, H(0) \geq 0, R(0) \geq 0$

Table 1: Description of the model parameters.

Parameter	Parameters description
b	Recruitment rate of the population
β	The rate of exposure of the susceptible to noise
α	Rate at which the exposed becomes affected
λ	Rate at which temporary hearing loss is regained or resolved
μ	Natural death rate
δ	The rate of death emanating from falls or accidents as a result of hearing loss
ϵ	Rate of re-exposure to noisy environment that could lead to further hearing loss

Also considering the following equation:

$$N = S + E + H + R \quad (2)$$

the derivative of N is given below with respect to t as

$$\frac{d}{dt}(S + E + H + R) = b - \mu N - \delta H \geq 0$$

which implies that

$$\frac{d}{dt}(S + E + H + R) \leq b - \mu N$$

It follows that

$$\lim_{t \to \infty} Sup(S + E + I + R) \leq \frac{b}{\mu}$$

Therefore, the feasible region of the system defined by Eq. (1) is given by

$$\Gamma = \{(S,\ E,\ H,\ R): S + E + H + R \leq \frac{b}{\mu},\ S > 0,\ E \geq 0,\ H \geq 0,\ R \geq 0\}$$

2.2 Basic reproduction number

The basic reproduction number is known to be an effective tool that predicts how different parameters affect the transmission/impact of the disease among the population, and it is often denoted by R_0. By using the next generation matrix, we obtain the basic reproduction number of Eq. (1) as follows:

Let $x^I = \begin{bmatrix} E \\ H \end{bmatrix},\ x^N = \begin{bmatrix} S \\ R \end{bmatrix}$ *and*

$$\frac{d}{dt}\begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} \beta SH \\ 0 \end{bmatrix} - \begin{bmatrix} -\epsilon R + (\alpha + \mu)E \\ -\alpha E + (\lambda + \mu + \delta)H \end{bmatrix}$$

then $F(S, E, H, R) = \begin{bmatrix} \beta SH \\ 0 \end{bmatrix}, V(S, E, H, R) = \begin{bmatrix} -\epsilon R + (\alpha + \mu)E \\ -\alpha E + (\lambda + \mu + \delta)H \end{bmatrix}$

The noise-free equilibrium point of Eq. (1) is equal to $\left(\frac{b}{\mu}, 0,0,0\right)$ (details in Section (3.1))

$$Z = \left(\frac{\partial F}{\partial(E, H)}\right)\Bigg|_{\left(\frac{b}{\mu},0,0,0\right)} = \frac{\partial}{\partial(E, H)}\begin{bmatrix} \beta HS \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \beta S \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\beta b}{\mu} \\ 0 & 0 \end{bmatrix},$$

$$W = \left(\frac{\partial_V}{\partial(E, H)}\right)\Bigg|_{\left(\frac{b}{\mu},0,0,0\right)} = \begin{bmatrix} (\alpha + \mu) & 0 \\ -\alpha & (\lambda + \mu + \delta) \end{bmatrix} \ and$$

$$W^{-1} = \frac{1}{(\alpha+\mu)(\lambda+\mu+\delta)}\begin{bmatrix} \lambda + \mu + \delta & 0 \\ \alpha & \alpha + \mu \end{bmatrix}.$$

Therefore

$$ZW^{-1} = \frac{1}{(\alpha + \mu)(\lambda + \mu + \delta)}\begin{bmatrix} \frac{\beta\alpha b}{\mu} & \frac{\beta b(\lambda + \mu + \delta)}{\mu} \\ 0 & 0 \end{bmatrix}$$

and the reproduction number is given by the spectral radius of ZW^{-1}, that is

$$R_0 = \frac{\beta\alpha b}{\mu(\alpha+\mu)(\lambda+\mu+\delta)} \quad (3)$$

3.Stability Analysis of the Model

In analysing the stability of the various steady states of the model equation, the Jacobian matrix of Eq. (1) is obtained as

$$J(S, E, H, R) = \begin{bmatrix} -\beta H - \mu & 0 & -\beta S & 0 \\ \beta H & -(\alpha + \mu) & \beta S & \epsilon \\ 0 & \alpha & -(\lambda + \mu + \delta) & 0 \\ 0 & 0 & \lambda & -(\mu + \epsilon) \end{bmatrix} \quad (4)$$

3.1Noise-free steady state

This entails a case where no individual is exposed to noise hazard. It therefore implies that the noise-free steady state exist when $E = 0$ which means $H = 0$ and $R = 0$. Consequently from

$$\frac{dS}{dt} = b - \beta SH - \mu S = 0$$

$$\Rightarrow b - \mu S^0 = 0 \quad \Rightarrow \quad S^0 = \frac{b}{\mu}$$

Thus the system, Eq. (1) has a noise-free steady state $E_0 = \left(\frac{b}{\mu}, 0,0,0\right)$.

To determine the local stability of the noise-free steady state E_0 the Jacobian matrix in Eq. (4) is evaluated at E_0 which resulted to

$$J(E_0) = \begin{bmatrix} -\beta H - \mu & 0 & -\beta S & 0 \\ \beta H & -(\alpha + \mu) & \beta S & \epsilon \\ 0 & \alpha & -(\lambda + \mu + \delta) & 0 \\ 0 & 0 & \lambda & -(\mu + \epsilon) \end{bmatrix}$$

and the analysis can be summarised with the following theorem.

Theorem 1 *The noise-free steady state,* E_0 *is locally asymptotically stable If the basic reproduction number,* $R_0 < 1$ *and unstable if* $R_0 > 1$*. The value of* R_0 *is determined using Eq. (3).*

Proof. Considering the Jacobian matrix for the noise-free steady state, Eq. (5) and performing some row operations to obtain the eigenvalues as the diagonal entries of the matrix give

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -\frac{\beta b}{\mu} & 0 \\ 0 & -(\alpha + \mu) & \frac{\beta b}{\mu} & \epsilon \\ 0 & 0 & -(\lambda + \mu + \delta)(\alpha + \mu) + \frac{\alpha\beta b}{\mu} & \alpha\epsilon \\ 0 & 0 & 0 & -(\mu + \epsilon)\left[(\lambda + \mu + \delta)(\alpha + \mu) - \frac{\alpha\beta b}{\mu}\right] + \lambda\alpha\epsilon \end{bmatrix}$$

This gives $\lambda_1 = -\mu,\ \ \lambda_2 = -(\alpha + \mu),\ \ \lambda_3 = -(\lambda + \mu + \delta)(\alpha + \mu) + \frac{\alpha\beta b}{\mu}$ and

$$\lambda_4 = -(\mu+\epsilon)\left[(\lambda+\mu+\delta)(\alpha+\mu)-\frac{\alpha\beta b}{\mu}\right]+\lambda\alpha\epsilon$$
$$= -[(\mu+\epsilon)(\mu+\delta)(\alpha+\mu)+\mu\lambda(\alpha+\mu+\epsilon)][1-R_f]$$

where $R_f = \frac{\alpha\beta b(\mu+\epsilon)}{\mu(\mu+\epsilon)(\mu+\delta)(\alpha+\mu)+\mu^2\lambda(\alpha+\mu+\epsilon)}$ (see details in Eq.(6)) and $\lambda_3 < 0$ if

$$\frac{\alpha\beta b}{\mu(\lambda+\mu+\delta)(\alpha+\mu)} < 1 \quad \Rightarrow \quad R_0 < 1$$

Consequently, $\lambda_1 < 0, \lambda_2 < 0, \lambda_4 < 0$ since $R_f > 1$ gives the feasibility region of the noise-endemic state which implies that for the noise-free state $R_f < 1$ and $\lambda_3 < 0$ if $R_0 < 1$.

Hence, the noise-free steady state, E_0 is locally asymptotically stable if $R_0 < 1$ and unstable when $R_0 > 1$.

3.2 Endemic steady state

The noise-endemic steady state, E_* of the model equation, Eq. (1) is obtained as $E_* = (S^*, E^*, H^*, R^*)$ where

$$S^* = \frac{a_1}{a_2}, \quad H^* = \frac{ba_2-\mu a_1}{\beta a_1}, \quad E^* = \frac{(\lambda+\mu+\delta)(ba_2-\mu a_1)}{\beta\alpha a_1}, \quad R^* = \frac{\lambda(ba_2-\mu a_1)}{(\mu+\epsilon)\beta a_1}$$

and $a_1 = (\mu+\epsilon)(\alpha+\mu)(\mu+\delta)+\lambda\mu(\alpha+\mu+\epsilon)$, $a_2 = \beta\alpha(\mu+\epsilon)$.

The endemic state exists if and only if $H^* = \frac{ba_2-\mu a_1}{\beta a_1} > 0$ which gives the biological feasibility of the model. This implies that

$$R_f = \frac{ba_2}{\mu a_1} = \frac{\alpha\beta b(\mu+\epsilon)}{\mu(\mu+\epsilon)(\mu+\delta)(\alpha+\mu)+\mu^2\lambda(\alpha+\mu+\epsilon)} > 1.$$

Hence, the analysis of the endemic steady state gives the following result.

Theorem 2 *The noise-endemic steady state, E_* is locally asymptotically stable whenever is exist, that is when $R_f > 1$ but it transits to the noise-free state when If $R_0 < 1$.*

Proof. The Jacobian matrix for the endemic equilibrium is obtained from Eq. (4) as

$$J(E_*) = \begin{bmatrix} \frac{-ba_2}{a_1} & 0 & -\frac{\beta a_1}{a_2} & 0 \\ \frac{ba_2-\mu a_1}{a_1} & -(\alpha+\mu) & \frac{\beta a_1}{a_2} & \epsilon \\ 0 & \alpha & -(\lambda+\mu+\delta) & 0 \\ 0 & 0 & \lambda & -(\mu+\epsilon) \end{bmatrix}$$

Carrying out some row operations generated the eigenvalues as the diagonal entries of the following triangular matrix

$$J(E_*) = \begin{bmatrix} \frac{-ba_2}{a_1} & 0 & -\frac{\beta a_1}{a_2} & 0 \\ 0 & -(\alpha+\mu)\frac{ba_2}{a_1} & b\beta & \frac{\epsilon ba_2}{a_1} \\ 0 & 0 & \frac{-(\lambda+\mu+\delta)(\alpha+\mu)(ba_2)}{a_1}+b\beta\alpha & \frac{\epsilon ba_2\alpha}{a_1} \\ 0 & 0 & 0 & -(\mu+\epsilon)a_3+\lambda a_4 \end{bmatrix}$$

where $a_1 = (\mu+\epsilon)(\alpha+\mu)(\mu+\delta)+\lambda\mu(\alpha+\mu+\epsilon) > 0$, $a_2 = \beta\alpha(\mu+\epsilon) > 0$

$$a_3 = \frac{(\lambda+\mu+\delta)(\alpha+\mu)ba_2}{a_1} - b\beta\alpha$$
$$= \frac{(\lambda+\mu+\delta)(\alpha+\mu)ba_2 - b\beta\alpha a_1}{a_1} = \frac{b\beta\alpha^2\lambda\epsilon}{a_1} > 0$$
$$a_4 = \frac{\epsilon ba_2\alpha}{a_1} > 0$$

Hence, $\lambda_1 = \frac{-ba_2}{a_1} < 0$, $\lambda_2 = -(\alpha+\mu)\frac{ba_2}{a_1} < 0$,

$$\lambda_3 = \frac{-(\lambda+\mu+\delta)(\alpha+\mu)ba_2}{a_1} + b\beta\alpha = -a_3 < 0$$
$$\lambda_4 = -(\mu+\epsilon)a_3+\lambda a_4 = \frac{-(\mu+\epsilon)b\beta\alpha^2\lambda\epsilon}{a_1} + \frac{\lambda\epsilon b\beta\alpha^2(\mu+\epsilon)}{a_1} = 0$$

Consequently, the endemic state, E_* is always stable whenever it exists, that is when $R_f > 1$.

3.3 Global Stability

For higher-dimensional systems, different techniques could be used to prove the global stability of steady states. Lyapunov function is the most common one which are used to establish the global stability of steady states.

Theorem 3 *Suppose that $R_0 < 1$, then (6) the noise-free steady state, E_0 is globally asymptotically stable.*

Proof. We begin by creating a Lyapunov function, and take into account the $SEHRE$ model in the space of the first three variables (S, E, H) only. The noise-free steady states for the entire $SEHRE$ model must be globally stable if the noise-free steady state for the first three equations is stable, which is obvious, $R(t) \to 0$. Since we are interested in working with only the positive orthant it is sufficient to work with $\mathbb{R}^3_+$.

$$V = \kappa\left(S - S^0 - S^0 \ln\frac{S}{S^0}\right) + \frac{1}{\alpha+\mu}E + \frac{1}{\alpha}H, \quad (7)$$

where $\kappa > 0$ is to be determined and $S^0 = \frac{b}{\mu}$. First, it is easy to see that $V = 0$ at the noise-free steady state. To show that $V > 0$ for all $(S,E,H) \neq \left(\frac{b}{\mu}, 0, 0\right)$, it is sufficient to prove that $\kappa S^0\left(\frac{S}{S^0} - 1 - \ln\frac{S}{S^0}\right) > 0$. Hence, differentiating Eq. (7) gives

$$\frac{d}{dt}V = \kappa\left(1-\frac{S^0}{S}\right)S' + \frac{1}{\mu+\alpha}E' + \frac{1}{\alpha}H'$$
$$= 2\kappa b - \kappa\beta SH - \kappa\mu S - \frac{\kappa b^2}{\mu S} + \frac{\kappa\beta b}{\mu}H + \frac{\beta}{\mu+\alpha}SH - \frac{\lambda+\mu+\delta}{\alpha}H$$

Let $\kappa = \frac{1}{\mu+\alpha}$ then on simplification, the equation becomes

$$\frac{d}{dt}V = -\kappa b\left(\frac{b}{\mu S} + \frac{\mu S}{b} - 2\right) + \frac{\lambda+\mu+\delta}{\alpha}(R_0 - 1)H$$

Since $R_0 < 1$, the last term is negative. Now we have to show that the first term is also negative. If we set $a = b/(\mu S)$, then it is sufficient to show that this expression $a + 1/a - 2$ is positive for every $a > 0, a \neq 1$. Indeed,

$$a + \frac{1}{a} - 2 = \frac{a^2 - 2a + 1}{a} = \frac{(a-1)^2}{a} > 0.$$

Hence, we have $\frac{d}{dt}V < 0$ for all $(S, E, H) \neq (S^0, 0,0)$. Therefore, by Lyapunov's theorem, the noise-free equilibrium is globally asymptotically stable.

Lemma 1 *Suppose that $x_1, x_2, \dots, x_n$ are n positive numbers. Then their arithmetic mean is greater than or equal to their geometric mean. In particular,*

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \dots x_n}$$

Theorem 4 *Suppose that $R_0 > 1$, then the noise-endemic steady state, E_* is globally asymptotically stable.*

Proof. Similarly, only the first three components of the system defined by Eq. (1) are taken into consideration, (S, E, H). Suppose that they belong to the positive orthant $\mathbb{R}^3_+$. We define a Lyapunov function

$$V = \kappa_1\left(S - S^* - S^*\ln\frac{S}{S^*}\right) + \kappa_2\left(E - E^* - E^*\ln\frac{E}{E^*}\right) + \kappa_3\left(H - H^* - H^*\ln\frac{H}{H^*}\right)$$

where $\kappa_1 > 0, \kappa_2 > 0$, and $\kappa_3 > 0$ will be determined later. Note that $V = 0$ when $(S, E, H) = (S^*, E^*, H^*)$ and $V > 0$ otherwise; V is also radially unbounded. Next is to prove that $\frac{dV}{dt}$ is negative.

Differentiating V with respect to t and substituting the values of $\frac{dS}{dt}$, $\frac{dE}{dt}$, and $\frac{dH}{dt}$ gives

$$\frac{dV}{dt} = \kappa_1\left(1 - \frac{S^*}{S}\right)\frac{dS}{dt} + \kappa_2\left(1 - \frac{E^*}{E}\right)\frac{dE}{dt} + \kappa_3\left(1 - \frac{H^*}{H}\right)\frac{dH}{dt}$$

$$\Rightarrow \quad 2a \quad = \kappa_1\left(1 - \frac{S^*}{S}\right)(b - \beta SH - \mu S) + \kappa_2\left(1 - \frac{E^*}{E}\right)(\beta SH - (\alpha+\mu)E) + \kappa_3\left(1 - \frac{H^*}{H}\right)(\alpha E - (\lambda+\mu+\delta)H).$$

Since $b = \beta S^* H^* + \mu S^*$, expanding the equation yield

$$\frac{dV}{dt} = -\kappa_1\frac{(S-S^*)^2}{S} + \kappa_1\beta S^* H^* - \kappa_1\beta SH - \kappa_1\beta\frac{S^{*2}H^*}{S} + \kappa_1\beta S^* H + \kappa_2\beta SH - \kappa_2(\alpha+\mu)E - \kappa_2\beta\frac{E^*SH}{E} + \kappa_2(\alpha+\mu)E^* + \kappa_3\alpha E - \kappa_3(\lambda+\mu+\delta)H - \kappa_3\alpha\frac{H^*E}{H} + \kappa_3(\lambda+\mu+\delta)H^*.$$

Let $\kappa_1 = \kappa_2$ and also by multiplying and dividing the fractions with the steady state value give

$$\frac{dV}{dt} = -\kappa_1\frac{(S-S^*)^2}{S} + \kappa_1\beta S^* H^* - \kappa_1\beta\frac{S^{*2}H^*}{S} + \kappa_1\beta S^* H - \kappa_2(\alpha+\mu)E - \kappa_2\beta S^* H^*\frac{E^*SH}{ES^*H^*} + \kappa_2(\alpha+\mu)E^* + \kappa_3\alpha E - \kappa_3(\lambda+\mu+\delta)H - \kappa_3\alpha E^*\frac{H^*E}{HE^*} + \kappa_3(\lambda+\mu+\delta)H^*.$$

Since all fractional terms are negative and the non-fractional terms are positive they can be combined. First, we note that $\beta S^* H^* = (\alpha+\mu)E^*$ from the corresponding steady state equation since $\kappa_1 = \kappa_2$. Next, assuming the value of κ_3 such that $\kappa_3(\lambda+\mu+\delta)H^* = \kappa_2(\alpha+\mu)E^*$ gives

$$\kappa_3 = \kappa_2\frac{\alpha+\mu}{\alpha}$$

Therefore, extracting $\kappa_1\beta S^* H^*$ from all terms in the equation produce

$$\frac{dV}{dt} = -\kappa_1\frac{(S-S^*)^2}{S} + \kappa_1\beta S^* H^*\left[3 - \frac{S^*}{S} - \frac{E^*SH}{ES^*H^*} - \frac{H^*E}{HE^*}\right] + (\kappa_1\beta S^* - \kappa_3(\lambda+\mu+\delta))H + (\kappa_3\alpha - \kappa_2(\alpha+\mu))E$$

The last two terms in the formula above are zero since $\kappa_3 = \kappa_2(\alpha+\mu)/\alpha$.

For $\kappa_1 = \kappa_2 = 1$ it implies that

$$\kappa_3 = \frac{\alpha+\mu}{\alpha}.$$

According to this, the derivative of the Lyapunov function becomes

$$\frac{dV}{dt} = -\frac{(S-S^*)^2}{S} + \beta S^* H^*\left[3 - \frac{S^*}{S} - \frac{E^*SH}{ES^*H^*} - \frac{H^*E}{HE^*}\right].$$

It is clear that the first term in the above equation is negative unless $S = S^*$. It is then required to show that the second term is also negative. Let

$$x_1 = \frac{S^*}{S}, \quad x_2 = \frac{E^*SH}{ES^*H^*}, \quad x_3 = \frac{H^*E}{HE^*}.$$

Then, $x_1x_2x_3 = 1$ and according to Lemma (1), the arithmetic mean is larger than the geometric mean. Hence,

$$\frac{S^*}{S} + \frac{E^*SH}{ES^*H^*} + \frac{H^*E}{HE^*} \geq 3.$$

Therefore, the second term in the differentiating equation of the Lyapunov function is negative, and it is zero whenever $(S, E, H) = (S^*, E^*, H^*)$

$$\Rightarrow \quad \frac{dV}{dt} \leq 0.$$

Now we have to apply the Krasovkii-LaSalle theorem and consider the set where the Lyapunov function is equal to zero:

$$\mathcal{S} = \{x \in \mathbf{R}^n | V'(x) = 0\}.$$

It is easy to see that $\frac{dV}{dt} = 0$ if and only if

$$S = S^* \quad \text{and} \quad \frac{S^*}{S} + \frac{E^*SH}{ES^*H^*} + \frac{H^*E}{HE^*} = 3$$

Since $S = S^*$, then $\frac{dS}{dt} = 0$, and from Eq. (1), we can conclude that $H = H^*$. Therefore,

$$\frac{S^*}{S} + \frac{E^*SH}{ES^*H^*} + \frac{H^*E}{HE^*} = 3 \quad \Rightarrow \quad \frac{E^*}{E} + \frac{E}{E^*} = 2.$$

Consequently, this equality holds if and only if $E = E^*$. Hence, the set $\mathcal{S}$ consists of the singleton (S^*, E^*, H^*).

3.4 Sensitivity Analysis

The aim of sensitivity analysis is to determine qualitatively the parameter with the highest impact on the dynamics of the model. If a little change in a parameter's value causes significant change in the dynamics of the model then the parameter is said to be sensitive. To perform sensitivity analysis of a dynamical system, we suppose that the system has m compartments c_i for $i = 1,2, \dots, m$ and n parameters k_j for $j = 1,2, \dots, n$.

Representing the model balanced equations as a system of differential equations as in [(Martcheva, 2015), (Rahman et al., 2021)] gives

$$\frac{dc_i}{dt} = f_i(c, k)$$

where $c \in \mathbb{R}^m$ and $k \in \mathbb{R}^n$. Non-normalization, half-normalization and full-normalization are the techniques employed for analysing the sensitivity of the model.

The non-normalization is given by

$$S_{ij} = \frac{\partial c_i(t)}{\partial k_j}$$

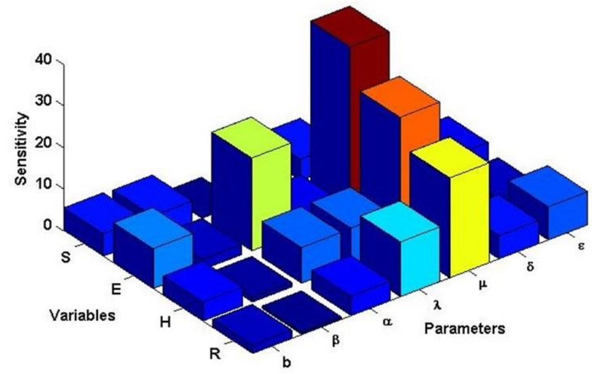
while the half-normalization is defined as

$$S_{ij} = \left(\frac{1}{c_i(t)}\right)\left(\frac{\partial c_i(t)}{\partial k_j}\right)$$

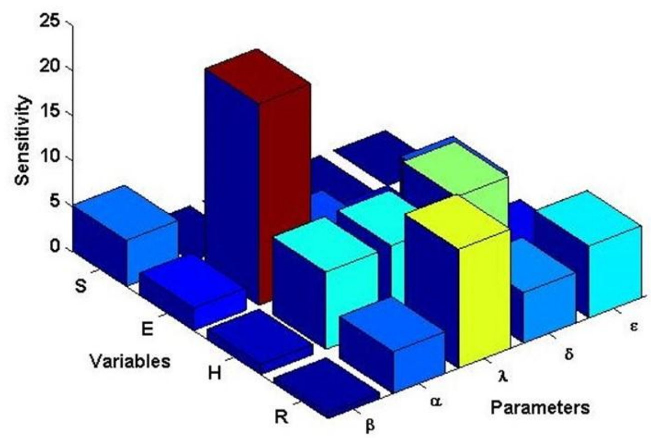
and the full-normalization is obtained as

$$S_{ij} = \left(\frac{k_j}{c_i(t)}\right)\left(\frac{\partial c_i(t)}{\partial k_j}\right)$$

where S_{ij} is the time-dependent sensitivities of c_i with respect to each parameter value k_j.

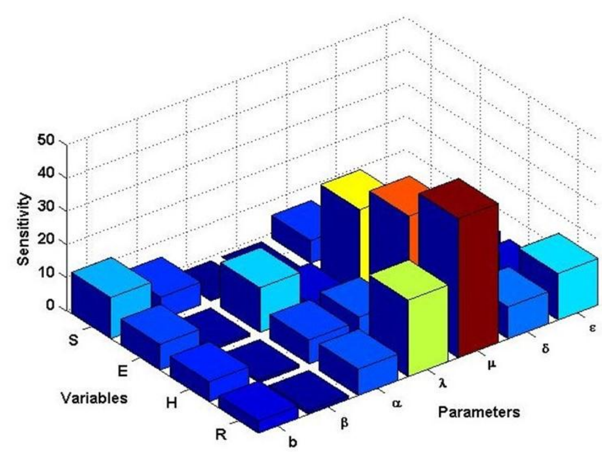


(a)



(b)

Figure 2: Local sensitivity analysis with non-normalization technique of all va[illegible]n computational simulations using [illegible]B with respect to (a) all parameters (b) with exception of b and μ.



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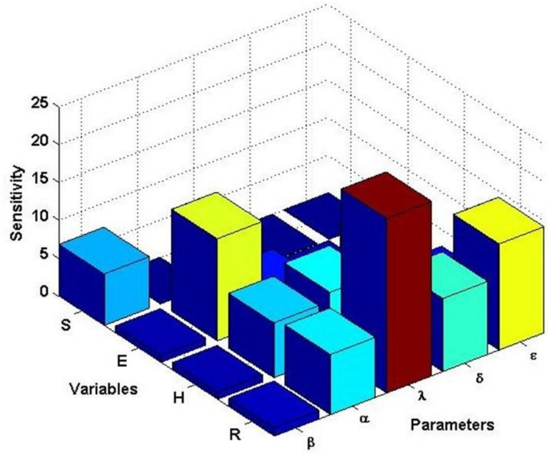

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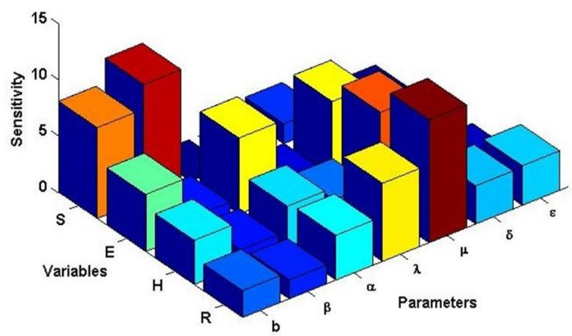
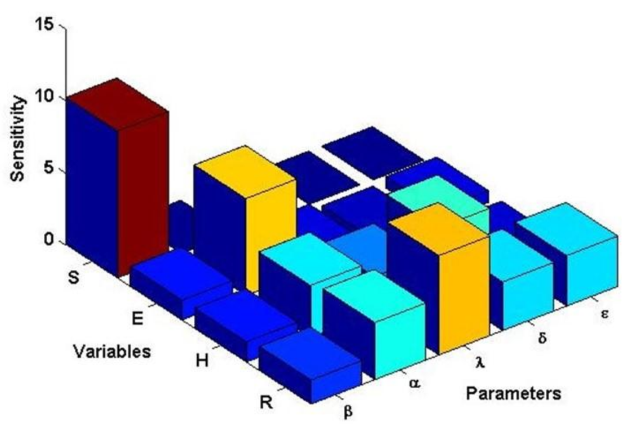
Figure 3: Local sensitivity analysis with half-normalization technique of all variables in computational simulations using MATLAB with respect to (a) all parameters (b) with exception of b and μ.



(a)



(b)

Figure 4: Local sensitivity analysis with full-normalization technique of all variables in computational simulations using MATLAB with respect to (a) all parameters (b) with exception of b and μ.

4. Numerical Simulation of the Model

In general, from the simulation results, it is observed that the various classes of the population are sensitive to the critical parameters. For instance, susceptible individuals are sensitive to β and b, whereas they exhibit less sensitivity with respect to δ and ϵ (see Fig. 4). In addition, Fig. 2 shows that the exposed individuals are sensitive to the model parameters μ and α while the affected individuals (those suffering loss of hearing) are sensitive to μ and δ. On the other hand, Fig. 3 shows that individuals having temporary hearing loss with chances of resolving it are sensitive to λ and μ as they have less sensitivity to parameter β.

Some contrasts and similarities are demonstrated based on the impact of each parameter used in the model steady states analysis with the three sensitivity analysis techniques. It is opined that, in comparison to other techniques, the full-normalization technique is more suitable for identifying the model critical parameters as observed in Fig. 4(a).

Furthermore, based on the basic reproduction number defined in Eq. (3), the stability

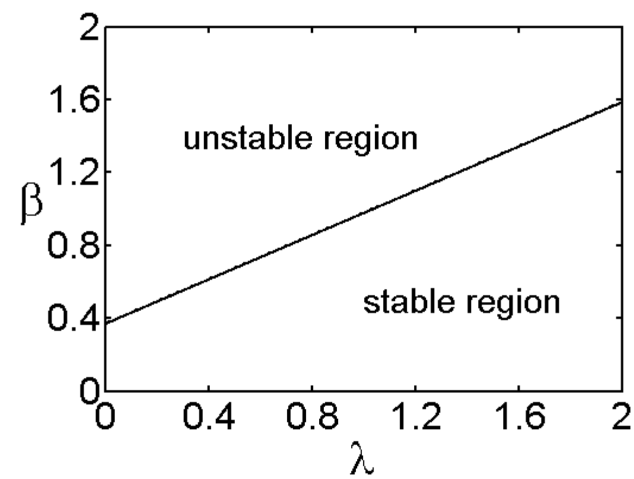
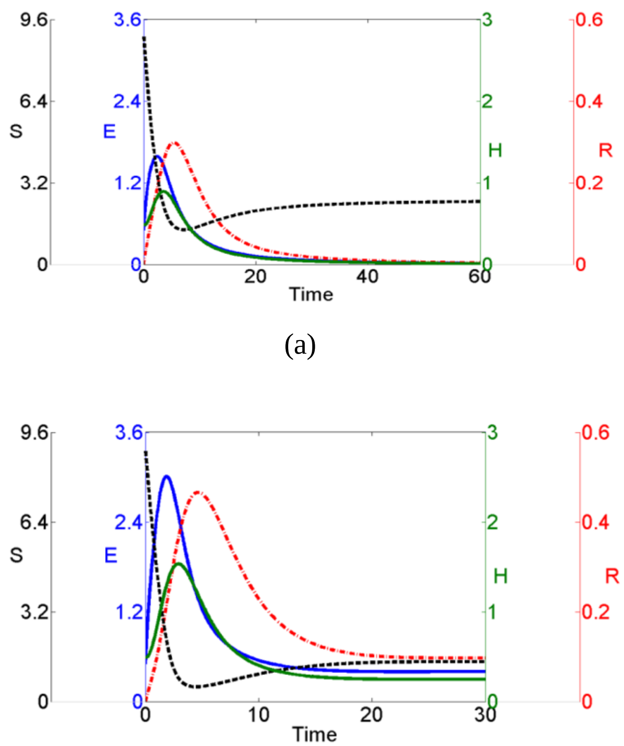


Figure 5: Stability region of hearing loss: The portions below the curves represent the unstable regions, while the portion above the curves represent the stable regions of the model.

Stability region of the model equation, Eq. (3) is obtained using MATLAB application and captured in Fig. 5. As shown in Fig. 5, the stability regions of the model indicate that the noise-free steady state increases with corresponding increase in the rate at which the temporary hearing loss is resolved or regain within the opportune period λ and/or decrease in the exposure rate to noise hazard, β. In contrast, the endemic steady state increases with corresponding increase in the exposure rate to noise hazard, β and/or decrease in the rate at which temporary hearing loss are resolved λ.

Consequently, the dynamics of the model equation for the noise-free and endemic steady states were obtained using the following set of parameter values: $b = 0.65, \delta = 0.35, \alpha = 0.5, \mu = 0.26\ and\ \epsilon = 0.25$ (see Fig. 6). For values of $R_0 < 1$, the dynamics of the model as captured by Fig. 6(a) indicate a noise-free steady state while for $R_0 > 1$

as shown in Fig. 6(b) the system transits to an endemic steady state. Hence, the outcome complement the results obtained earlier from the precious sections.



(a)

(b)

Figure 6: The dynamics of the model equation for (a) $R_0 < 1$, noise-free steady state (normal hearing) (b) $R_0 > 1$, endemic steady state (loss of hearing)

5. Discussion

In conclusion, this study presents a mathematical model of hearing loss caused by noise exposure using ordinary differential equations. The $SEHRE$ model divides the population into four compartments: susceptible individuals (S), exposed individuals (E), affected individuals (H), and individuals with temporary hearing loss that could be resolve within a given time duration (R). By analysing the model, the existence and stability of the noise steady states were established. The noise-free state was shown to be globally asymptotically stable if the basic reproductive number is less than one, while the endemic state was found to be globally asymptotically stable if the basic reproductive number is greater than one. This implies that the model is well-posed and can accurately describe the dynamics of hearing loss caused by noise exposure.

The sensitivity analysis of the critical parameters of the model highlights the importance of reducing exposure to noise in order to prevent this hearing loss. These findings have important implications for public health and policy decision-making, as they can inform interventions aimed at reducing noise exposure and preventing hearing loss. In summary, the mathematical model presented in study provides a valuable tool for understanding the dynamics of hearing loss caused by noise exposure, and can inform effective and targeted public health interventions to mitigate its impact. It is hoped that these findings will contribute to the development of more effective strategies for the prevention and management of hearing loss. In the future, we plan to improve our model by using real data, ideally collected from hospitals or workplaces. This will help us test how well the model works in real situations involving noise exposure and its effects on hearing loss.

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