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RESEARCH PAPER

Determination of the Astrophysical S-factor and Thermonuclear Reaction Rates of the (α,n) Medium Elements Reactions

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A B S T R A C T:

Cross-sections of the (α,n) medium elements reactions as a function of energies of alpha (α)-particle such as ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn , ^{50}Cr(α,n)^{53}Fe , ^{55}Mn(α,n)^{58}Co , ^{54}Fe(α,n)^{57}Ni , ^{59}Co(α,n)^{62}Cu , ^{62}Ni(α,n)^{65}Zn , ^{63}Cu(α,n)^{66}Ga , and ^{66}Zn(α,n)^{69}Ge have been interpolated from threshold to 10 MeV in step of 0.05 MeV by using the Program of MATLAB. Weighted averages of the Cross-sections in (mb) have been utilized to calculate the astrophysical S-factor and thermonuclear reaction rates as a function of the energy of the center of mass, $E_{c.m.}$ and T_9 Which is the temperature in units of $10^9 K$ ($T_9 = 10^{-9} T$)

respectively. Polynomial relationships have been utilized to fit the computed astrophysical S-factor and thermonuclear reaction rates to determine the astrophysical S-factor at various $E_{c.m.}$ and thermonuclear reaction rates at various T_9 from best fitting equations with the minimum Chi-Square. Empirical formulae of set of reactions ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn , ^{55}Mn(α,n)^{58}Co , ^{59}Co(α,n)^{62}Cu , and ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn , ^{55}Mn(α,n)^{58}Co , ^{62}Ni(α,n)^{65}Zn , ^{66}Zn(α,n)^{69}Ge have been utilized to compute astrophysical S-factor as a function of $E_{c.m.}$ and Z and thermonuclear reaction rates as a function of T_9 and the target nucleus atomic number Z. The results have been compared with the embraced astrophysical S-factor and thermonuclear reaction rates that have been calculated from the fitting equations which have a good agreement.

KEY WORDS: Cross-sections; astrophysical S-factor; thermonuclear reaction rates; Gamow factor; Gamow energy; Sommerfeld parameter.
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INTRODUCTION :

The astrophysical S-factor, S(E), has covered a large area which used in the field to remove the energy dependence of the Coulomb barrier penetration from the cross-section, $\sigma(E)$ (Jose, 2016). As stellar energies are much lower than the Coulomb barrier, the cross sections hardly depend on energy (Descouvemont, 2011).

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Thermonuclear reactions play an important role in supplying the major source of energy in stars in particular during hydrogen burning. This burning process in the stellar interiors consists of the proton-proton (pp) chain and the carbon-nitrogen-oxygen (CNO) cycle (Abdul Aziz, 2008). The quantity of interest in computing thermonuclear reaction rates for astrophysical aims is $N_A < \sigma v >$, which is the product of Avogadro's number with the average value of the cross section times velocity, averaged over a Maxwell-Boltzmann distribution of temperature (Roughton *et al.*, 1983). Total Cross-sections of the (α,n) medium element reactions, that is a function of center of mass energy, have been calculated by a few authors, which are reminded by various references such as ^{45}Sc(α,n)^{48}V(Vlieks, Morgan and Blatt, 1974; Hansper *et al.*, 1989; Haider, 2012) , ^{48}Ti(α,n)^{51}Cr(Chang *et al.*, 1973; Vonach, Haight and Winkler, 1983; Levkovski, 1991; Morton *et al.*, 1992; Baglin, Coral *et al.*, 2004), ^{51}V(α,n)^{54}Mn (Levkovski, 1991; Hansper *et al.*, 1993; Sonzogni *et al.*, 1993; Peng, He and Long, 1999; Noori, 2008; Haider, 2012), ^{50}Cr(α,n)^{53}Fe(Vlieks, Morgan and Blatt, 1974; Morton *et al.*, 1994; Haider, 2012) , ^{55}Mn(α,n)^{58}Co(Rizvi *et al.*, 1989; Levkovski, 1991; Tims *et al.*, 1993; Haider, 2012) , ^{54}Fe(α,n)^{57}Ni (Houck and Miller, 1961; Vlieks, Morgan and Blatt, 1974; Tims *et al.*, 1991; Haider, 2012) , ^{59}Co(α,n)^{62}Cu(Stelson and McGowan, 1964; D`auria *et al.*, 1968; Zhukova *et al.*, 1972; Tims *et al.*, 1988; Noori, 2008) , ^{62}Ni(α,n)^{65}Zn (Stelson and McGowan, 1964; Levkovski, 1991; Haider, 2012) , ^{63}Cu(α,n)^{66}Ga(Stelson and McGowan, 1964; Zhukova *et al.*, 1970; Haider, 2012) , and ^{66}Zn(α,n)^{69}Ge(Stelson and McGowan, 1964; Levkovski, 1991) respectively. The goal of this work is to determine the empirical formulae to compute the astrophysical S-factor, S(E), and thermonuclear reaction rates, $N_A\langle\sigma v\rangle$, utilizing the altered cross- sections of the reactions of the medium elements. The outcomes were compared with those published in the previous work.

2. Theory

Atomic masses of each medium element and isotopes related to this present work have been taken from the nuclear wallet cards published by the National Nuclear Data Center (NNDC) (Tuli, 2011). The Q-Value of the reaction $X(\alpha,n)Y$, is defined as the difference between the initial and the final rest mass energies (Meyerhof, 1967):

$$Q = [M_\alpha + M_X - (M_Y + M_n]c^2 \quad (1)$$

Where (M_α , M_X , M_Y , and M_n) are the atomic masses of the incident, target particles, product nucleus and neutron (outgoing particle), respectively and (c^2 = 931.494013 MeV/u; where u=atomic mass unit (amu) =1.66×10^{-27} kg). This equation is called the Q-value equation. If Q is + ive, the reaction called exoergic; if Q is - ive, it is endoergic.

The amount of energy needed for an endoergic reaction is called the *threshold energy* and can be calculated easily (Kaplan, 1962).

$$E_{th} = -Q(1 + \frac{M_\alpha}{M_X}) \quad (2)$$

Fusion requires two (or more) interacting particles to approach closely enough, within the short range of the (attractive) strong nuclear force, $\lesssim 10^{-15}$ m, to construct a new nucleus with A = A_1+A_2.
The so-called height V_C of the barrier is its maximum value, which occurs at the nuclear radius, and is (Evans, 1955).

$$V_C = \frac{Z_1 Z_2 e^2}{R} \quad (3)$$

Where $Z_1$and Z_2 are the charges of the projectile and target nuclei, and R and $(R = R_1 + R_2)$ is their separation, e is the charge of electron ($e^2 =$ 1.44 $MeV\ fm$), and the radius of the nucleus is given by $R = 1.3 \times 10^{-13} A^{1/3}$ cm, where A is the mass number (atomic weight) (Shaviv, 2012). Then Eq. (3) leads to

$$V_C = E_C = \frac{1.44}{1.3}\left(\frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}}\right) \quad (4)$$

Where E_C is the coulomb barrier (Coulomb energy) in MeV, $A_1^{1/3}$ and $A_2^{1/3}$ are the mass numbers of the charges of bombarding and targeting nuclei respectively.

The astrophysical S-factor, $S(E)$, in the unit (MeV-b) is related to the cross-section by (Li, J. *et al.*, 2012):

$$S(E) = E\sigma(E)\exp(2\pi\eta) \quad (5)$$

Where E is the center-of-mass energy ($E_{c.m.}$) in MeV, $\sigma(E)$ is the cross-section of the reaction in (mb), $2\pi\eta$ is the Gamow factor, and η is Sommerfeld parameter (Angulo *et al.*, 1999):

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = 0.1575 Z_1 Z_2 \sqrt{\frac{\mu(u)}{E(MeV)}} \quad (6)$$

, $\hbar$ is Planck's constant over 2π (1. 0546 × 10^{-27} ergs), v is the relative velocity, μ is the reduced mass. The Gamow factor G (E) or $2\pi\eta$ can be written as in (Jose, 2016):

$$2\pi\eta = 0.98951 Z_1 Z_2 \sqrt{\frac{\mu(u)}{E(MeV)}} \quad (7)$$

The reduced mass μ in u (amu) is determined by the relationship (Clayton, 1968):

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad (8)$$

Where $m_1$and m_2 represent the masses of the bombarding and target nucleus in units of (amu), respectively. The energy of the center of mass of pair of particles $E_{c.m.}$ is related to the laboratory energy, $E_{Lab.}$ of the projectile particle by the equation (Meyerhof, 1967):

$$E_{c.m.} = \frac{m_2}{m_1 + m_2} E_{lab.} \qquad (9)$$

The Gamow energy E_G , in MeV (Brown, 2015):

$$E_G = 2\pi^2 \mu u C^2 \alpha^2 (Z_1 Z_2)^2 = 0.979\mu (Z_1 Z_2)^2 \qquad (10)$$

Where $\alpha = \frac{1}{137} = \frac{e^2}{\hbar c}$ is the fine-structure constant. The thermonuclear reaction rates, $N_A\langle\sigma v\rangle$ in unit $(cm^3 mol^{-1} s^{-1})$ (Angulo *et al.*, 1999):

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(k_B T)^{3/2}} N_A \int_0^\infty E\sigma(E) \exp(-E/k_B T)\, dE \qquad (11)$$

Where N_A is the Avogadro's number ($6.022 \times 10^{23} mol^{-1}$), k_B is the Boltzmann's constants $(1.38 \times 10^{-16} erg/K)$, and $T\ is\ the\ temperature$ respectively. Eq. (11) leads to (Angulo *et al.*, 1999):

$$N_A\langle\sigma v\rangle = 3.7313 \times 10^7 \mu^{-1/2} T_9^{-3/2} \int_0^\infty E\sigma(E) \exp(-11.605E/T_9)\, dE \qquad (12)$$

Where T_9 is the temperature in units of $10^9 K$ ($T_9 = 10^{-9} T$)

The weighted averages of the Cross-sections of medium elements $\sigma_0(mb)$ and the uncertainty (errors) $\Delta\sigma_0(mb)$ are expressed by the following Eqs. (Bevington and Robinson, 2003):

$$\sigma_0(mb) = \frac{\sum_i (\sigma_i / {\delta_i}^2)}{\sum_i (1/{\delta_i}^2)} \qquad (13)$$

Where σ_i and δ_i $(\Delta\sigma_i)$ are the cross-section and the uncertainties of i^{th} reference, relating to each value of σ_i,

$$\Delta\sigma_0(mb) = \pm \frac{1}{\sqrt{\sum_i (1/\delta_i^2)}} \qquad (14)$$

The considered formalism type is the polynomial fit expression of the shape:

$$Y = C_0 + C_1 X + C_2 X^2 + C_3 X^3 + \cdots + C_N X^N = \sum_{i=0}^{M} C_i X^i \qquad (15)$$

This polynomial is obtained by the Excel computer program (Format Trendline). Where $(C_0, C_1, C_2, C_3, \dots)$ are free parameters (coefficients of polynomial), and $(i = 0, 1, 2, 3, \dots, M)$, and

$$C_i = \sum_{j=0}^{N} C_{ij} K^j \qquad (16)$$

Are considered in this work, then by combining the Eqs. (15) & (16), the following relation has been acquired:

$$Y = \sum_{i=0}^{M} (\sum_{j=0}^{N} C_{ij} K^j) X^i \qquad (17)$$

Where Y=ln[S(E)] or ln[N_A<σv>], (i=0,1,2,…M), (j=0,1,2,…N), (C_{00},C_{01},C_{02},….) are coefficients of polynomials, K is the energy of the center of mass or T_9 according to the S(E) or N_A<σv>, and X is atomic number Z. The Excel computer program has been utilized to acquire the best fit relationship corresponding to various energies ranges near threshold up to 10 MeV in the center of mass system or T_9 ranges from (1 to 10) 10^9K. The data of these extents were avoided in each step, till a possible value of the determination coefficient $R^2 \approx 1$ was come to. The best fit adopted data was acquired with increasing order to supply the minimum value of Chi-Square (χ^2) by using the Eq. (Belgaid *et al.*, 2005):

$$\chi^2 = \frac{1}{(N-M)} \sum_i^N \left(\frac{Y_{exp}^i - Y_{cal}^i}{\Delta Y_{exp}^i}\right)^2 \qquad (18)$$

Where N is the data points' number, M is the fitting coefficients number, Y_{exp}^i and ΔY_{exp}^i are the experimental (adopted value) of $ln[S(E)]$ or $ln[N_A\langle\sigma v\rangle]$ and its error (uncertainty) respectively, Y_{cal}^i is the calculated $ln[S(E)]$ or $ln[N_A\langle\sigma v\rangle]$.

3. Data Reduction and Analysis

The Atomic masses have been taken into consideration to determine the Q-Value, threshold energy, Coulomb barrier, reduced mass, and the ratio between ($E_{c.m.}$/$E_{lab.}$) of (α, n) medium elements reactions using the Eqs. (1, 2, 4, 8, and 9); the results have been shown in the table (1). Eqs. (6,7,10, and 5) taken into consideration to determine the Sommerfeld parameter(η), Gamow factor G(E), Gamow energy (E_G), and the S-factor of astrophysical, S(E) of the (α,n) medium element reactions. The results are shown in table (2). The cross-sections of (α,n) reactions of medium elements in present work such as (^{45}Sc, ^{48}Ti, ^{51}V, ^{50}Cr, ^{55}Mn, ^{54}Fe, ^{59}Co, ^{62}Ni, ^{63}Cu, and ^{66}Zn), which are available in the literature review has been taken and plotted again, and using the MATLAB software to interpolate to acquire the cross-sections in fine step of 0.05 MeV. The weighted average of the altered Cross-sections of i^{th} references for the medium elements which cross-section (σ_0) and uncertainty ($\Delta\sigma_0$) have been computed by using Eqs. (13) and (14) respectively,. The acquired results have been utilized to calculate the astrophysical S-factor and thermonuclear reaction rates of (α,n) reactions as a function of the center of mass energies $E_{c.m.}$ by using eq. (5) and (12). The acquired equations to compute the S-factor of the reminded reactions are shown in Table 2.

The final formula for each astrophysical S-factor, $S(E)$ and thermonuclear reaction rates $N_A\langle\sigma v\rangle$ is shown in Eq. (17) where $Y = \ln[\text{S(E)}]$ or $Y = \ln[N_A\langle\sigma v\rangle]$.

Table 1. Q-Value, threshold energy ($E_{threshold}$), Coulomb barrier E_c, reduced mass ($\boldsymbol{\mu}$), and the ratio between ($E_{c.m.}/E_{lab.}$) of (α, n) medium elements reactions.

(α,n) Medium Element Reaction	Q-value (MeV)	E threshold (MeV)		Coulomb Barrier E_c (MeV)	Reduced Mass (μ) (amu)	Ec.m./Elab.
		Lab. System	C.M. System			
^{45}Sc(α,n)^{48}V	-2.241E+00	2.440E+00	2.241E+00	9.044E+00	3.675E+00	9.182E-01
^{48}Ti(α,n)^{51}Cr	-2.687E+00	2.911E+00	2.687E+00	9.334E+00	3.694E+00	9.230E-01
^{51}V(α,n)^{54}Mn	-2.294E+00	2.474E+00	2.294E+00	9.622E+00	3.711E+00	9.272E-01
^{50}Cr(α,n)^{53}Fe	-4.961E+00	5.359E+00	4.961E+00	1.009E+01	3.706E+00	9.258E-01
^{55}Mn(α,n)^{58}Co	-3.512E+00	3.767E+00	3.512E+00	1.027E+01	3.731E+00	9.321E-01
^{54}Fe(α,n)^{57}Ni	-5.817E+00	6.249E+00	5.817E+00	1.073E+01	3.726E+00	9.309E-01
^{59}Co(α,n)^{62}Cu	-5.089E+00	5.434E+00	5.089E+00	1.091E+01	3.748E+00	9.364E-01
^{62}Ni(α,n)^{65}Zn	-6.480E+00	6.899E+00	6.480E+00	1.119E+01	3.760E+00	9.393E-01
^{63}Cu(α,n)^{66}Ga	-7.502E+00	7.979E+00	7.502E+00	1.154E+01	3.763E+00	9.402E-01
^{66}Zn(α,n)^{69}Ge	-7.445E+00	7.897E+00	7.445E+00	1.181E+01	3.774E+00	9.428E-01

Table 2. The Sommerfeld parameter(η), Gamow factor G(E), Gamow energy (E_G), and the astrophysical S-factor, S(E) of the (α,n) medium elements reactions

(α,n) Medium Element Reaction	Sommerfeld Parameter η	Gamow factor G(E)	Gamow Energy Eg(MeV)	Astrophysical S-factor S(E)
^{45}Sc(α,n)^{48}V	$1.268E+01/\sqrt{E_{c.m.}}$	$7.967E+01/\sqrt{E_{c.m.}}$	6.348E+03	$E_{c.m.}\sigma(E)Exp(7.967E+01/\sqrt{E_{c.m.}})$
^{48}Ti(α,n)^{51}Cr	$1.331E+01/\sqrt{E_{c.m.}}$	$8.368E+01/\sqrt{E_{c.m.}}$	7.003E+03	$E_{c.m.}\sigma(E)Exp(8.368E+01/\sqrt{E_{c.m.}})$
^{51}V(α,n)^{54}Mn	$1.395E+01/\sqrt{E_{c.m.}}$	$8.769E+01/\sqrt{E_{c.m.}}$	7.689E+03	$E_{c.m.}\sigma(E)Exp(8.769E+01/\sqrt{E_{c.m.}})$
^{50}Cr(α,n)^{53}Fe	$1.455E+01/\sqrt{E_{c.m.}}$	$9.143E+01/\sqrt{E_{c.m.}}$	8.360E+03	$E_{c.m.}\sigma(E)Exp(9.143E+01/\sqrt{E_{c.m.}})$
^{55}Mn(α,n)^{58}Co	$1.520E+01/\sqrt{E_{c.m.}}$	$9.556E+01/\sqrt{E_{c.m.}}$	9.132E+03	$E_{c.m.}\sigma(E)Exp(9.556E+01/\sqrt{E_{c.m.}})$
^{54}Fe(α,n)^{57}Ni	$1.580E+01/\sqrt{E_{c.m.}}$	$9.932E+01/\sqrt{E_{c.m.}}$	9.865E+03	$E_{c.m.}\sigma(E)Exp(9.932E+01/\sqrt{E_{c.m.}})$
^{59}Co(α,n)^{62}Cu	$1.646E+01/\sqrt{E_{c.m.}}$	$1.034E+02/\sqrt{E_{c.m.}}$	1.070E+04	$E_{c.m.}\sigma(E)Exp(1.034E+02/\sqrt{E_{c.m.}})$
^{62}Ni(α,n)^{65}Zn	$1.709E+01/\sqrt{E_{c.m.}}$	$1.074E+02/\sqrt{E_{c.m.}}$	1.154E+04	$E_{c.m.}\sigma(E)Exp(1.074E+02/\sqrt{E_{c.m.}})$
^{63}Cu(α,n)^{66}Ga	$1.771E+01/\sqrt{E_{c.m.}}$	$1.113E+02/\sqrt{E_{c.m.}}$	1.240E+04	$E_{c.m.}\sigma(E)Exp(1.113E+02/\sqrt{E_{c.m.}})$
^{66}Zn(α,n)^{69}Ge	$1.835E+01/\sqrt{E_{c.m.}}$	$1.153E+02/\sqrt{E_{c.m.}}$	1.330E+04	$E_{c.m.}\sigma(E)Exp(1.153E+02/\sqrt{E_{c.m.}})$

4. Results and Discussion

In general, we can write Eq. (15), and instead of X insert center of mass energies $E_{c.m.}$. Then the Eq. (15) becomes

$$Y = C_0 + C_1K + C_2K^2 + C_3K^3 + \cdots + C_NK^N = \sum_{i=0}^{M} C_iK^i \quad (19)$$

Where (C_0, C_1, C_3 …) are free parameters, K are parameters that represent the C.M energy or T_9, (i=0, 1, 2, 3… M), and Y=ln[S-factor (MeV-b)] or Y=ln[N_A<σv> ($cm^3mol^{-1}s^{-1}$)].

4.1. Astrophysical S-factor Empirical Formulae

The adopted astrophysical S-factor has been used to acquire the fitting parameters by using the expressions of the polynomial (18), (20) and (19) as shown in the steps:

1. The polynomial relations which are utilized in eq. (19) to fit the computed astrophysical S-factor, S(E) in the natural logarithm of the calculated elements to compute the adopted (taken on) natural logarithm of astrophysical S-factor from the best fitting with a minimum (χ^2) using Eq. (20). The acquired best fitting relations of the reminded reactions were presented in Eqs. (20, 21, 22, 23, 24, 25, 26, 27, 28, and 29) for the reactions ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn , ^{50}Cr(α,n)^{53}Fe , ^{55}Mn(α,n)^{58}Co , ^{54}Fe(α,n)^{57}Ni , ^{59}Co(α,n)^{62}Cu , ^{62}Ni(α,n)^{65}Zn , ^{63}Cu(α,n)^{66}Ga , and ^{66}Zn(α,n)^{69}Ge respectively.

^{45}Sc(α,n)^{48}V $x^2 = 0.0247$

$$ln[S - factor(MeV - b)] = 0.0062E^3 - 0.2075E^2 + 1.0183E + 30.7 \quad (20)$$

^{48}Ti(α,n)^{51}Cr $x^2 = 0.086$

$$ln[S - factor\ (MeV - b)] = -0.0867E^4 + 2.5599E^3 - 28.125E^2 + 135.15E - 206.65 \quad (21)$$

^{51}V(α,n)^{54}Mn $x^2 = 0.041$

$$ln[S - factor\ (MeV - b)] = -0.0261E^3 + 0.4478E^2 - 3.1956E + 41.906 \quad (22)$$

^{50}Cr(α,n)^{53}Fe $x^2 = 0.682$

$$ln[S - factor\ (MeV - b)] = 0.0763E^3 - 1.9295E^2 + 15.185E - 5.9137 \quad (23)$$

^{55}Mn(α,n)^{58}Co $x^2 = 0.0015$

$$ln[S - factor\ (MeV - b)] = 0.0153E^3 - 0.4515E^2 + 3.3053E + 28.436 \quad (24)$$

^{54}Fe(α,n)^{57}Ni $x^2 = 0.051$

$$ln[S - factor\ (MeV - b)] = 0.2583E^3 - 6.0359E^2 + 45.678E - 78.034 \quad (25)$$

^{59}Co(α,n)^{62}Cu $x^2 = 0.112$

$$ln[S - factor\ (MeV - b)] = 0.0872E^3 - 2.205E^2 + 17.564E - 8.1943 \quad (26)$$

^{62}Ni(α,n)^{65}Zn $x^2 = 0.0056$

$$ln[S - factor\ (MeV - b)] = 0.1946E3E^3 - 4.9013E^2 + 40.02E - 69.093 \quad (27)$$

^{63}Cu(α,n)^{66}Ga $x^2 = 0.027$

$$ln[S - factor\ (MeV - b)] = 2.2587E^3 - 57.223E^2 + 480.3E - 1299.5 \quad (28)$$

^{66}Zn(α,n)^{69}Ge x^2= 0.685E-03

$$ln[S - factor\ (MeV - b)] = 0.7816E^3 - 20.777E^2 + 183.26E - 498.11 \quad (29)$$

2. At fixed values of energy in center-of-mass, the change of the S-factor in natural logarithm with the Z has been fitted to the polynomial relation utilizing Eq. (19). The acquired results were used to determine the free parameters (coefficients of polynomial) (C_i).
3. The free parameters C_i, were plotted against each value of the center of mass energies and fitted to sufficient the polynomial relation were shown in Eq. (16).
4. The last formula of a set of reactions has been calculated by utilizing the combination of the two polynomials to show the systematic manner of the reactions which are shown in Eq. (17). The Y Variable is the astrophysical S-factor.

4.1.1 The Empirical Formulae Relating the Astrophysical S-factor to Center of Mass Energy and the Atomic Number Z of the Target Nucleus

The empirical formulae related to the astrophysical S-factor (MeV-b) with both of center of mass energy $E_{c.m.}$, and the atomic number Z were performed as the steps below:

1- At fixed values of the center of mass energies from 5.5 to 10 MeV in steps of 0.25 MeV for the ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn ,^{55}Mn(α,n)^{58}Co , and ^{59}Co(α,n)^{62}Cu reactions, the astrophysical S- factor in natural logarithm will vary with the atomic number(Z), as shown in Fig. (1). The data was fitted into the accompanying polynomial expression:

$$Y = \sum_{i=0}^{2} C_iX^i \quad (30)$$

Where $Y = \ln[S(E)]$, and X=Z, with free parameters C_i (C_0, C_1, and C_2).

2- The S-factor, S(E), which is was adopted, has been utilized as a function of atomic number Z of target nucleus at the fixed center of mass energies using the computer program Excel to acquire the fitting relations and then it was utilized to compute the fitting parameters. The acquired results are shown in Table 3.

3- The obtained free parameters C_i (C_0, C_1, and C_2), presented in Table (3) are plotted against with the fixed values of center of mass energies from 5.5 to 10 MeV in step of 0.25 MeV as shown in Fig.(2), and then the acquired coefficients of polynomials C_i have been fitted to the polynomial expression below:

$$C_i = \sum_{j=0}^{2} C_{ij} E^j \qquad (31)$$

The combination of the two polynomials Eq. (30) and Eq. (31) takes the form of the formula below of energy ranged from 5.5 to 10 MeV in the step of 0.25 MeV:

$$Y = \sum_{i=0}^{2} \left(\sum_{j=0}^{2} C_{ij} E^j\right) X^i \qquad (32)$$

Where Y=ln[S(E)], X=atomic number Z

$$Y = \sum_{i=0}^{2} (C_{i0}E^0 + C_{i1}E^1 + C_{i2}E^2) X^i$$

$$Y = C_{00}E^0X^0 + C_{01}E^1X^0 + C_{02}E^2X^0 + C_{10}E^0X^1 + C_{11}E^1X^1 + C_{12}E^2X^1 + C_{20}E^0X^2 + C_{21}E^1X^2 + C_{22}E^2X^2 \qquad (33)$$

Where (C_{00}, C_{01}, C_{02}, C_{10}, C_{11}…. C_{22}) are free parameters and their values are shown in the matrix below:

$$\begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} -2.0079 & -14.163 & 1.4295 \\ 2.5083 & 1.1719 & -0.1305 \\ -0.0466 & -0.0217 & 0.0027 \end{bmatrix}, \begin{bmatrix} R^2 = 0.6757 \\ R^2 = 0.7426 \\ R^2 = 0.7585 \end{bmatrix}$$

The acquired formula of a set of reactions such as $^{45}Sc(\alpha,n)^{48}V$, $^{48}Ti(\alpha,n)^{51}Cr$, $^{51}V(\alpha,n)^{54}Mn$, $^{55}Mn(\alpha,n)^{58}Co$, and $^{59}Co(\alpha,n)^{62}Cu$ has been used to calculate the astrophysical S-factor S(E) for each of the above reactions and compared with the adopted astrophysical S-factor calculated from the fitting expressions and shown to be in a good agreement and the comparison of the two results are shown in Table (4).

Table 3. Free parameters C_i (C_0, C_1, and C_2) as a function of the energy of the center of mass .

Ec.m. (MeV)	C0	C1	C2
5.5	-49.997	6.1814	-0.1106
5.75	-41.303	5.3931	-0.093
6	-34.449	4.7574	-0.0787
6.25	-29.331	4.2685	-0.0675
6.5	-25.812	3.9172	-0.0593
6.75	-23.719	3.6906	-0.0538
7	-22.843	3.5724	-0.0508
7.25	-22.944	3.5423	-0.0497
7.5	-23.743	3.5767	-0.0501
7.75	-24.928	3.6484	-0.0515
8	-26.154	3.7264	-0.053
8.25	-27.038	3.7763	-0.054
8.5	-27.164	3.7599	-0.0535
8.75	-26.081	3.6355	-0.0508
9	-23.304	3.3579	-0.0446
9.25	-18.311	2.8781	-0.0339
9.5	-10.548	2.1436	-0.0176
9.75	0.5769	1.0983	0.0057
10	15.687	-0.3175	0.0374

Table 4. Comparison between polynomial fitting expression (Best Fitting) of the adopted astrophysical S-Factor of (α,n) medium element reactions with those computed from Eq. (33).

Ec.m	$^{45}Sc(\alpha,n)^{48}V$	$^{48}Ti(\alpha,n)^{51}C$	$^{51}V(\alpha,n)^{54}Mn$	$^{55}Mn(\alpha,n)^{58}Co$	$^{59}Co(\alpha,n)^{62}Cu$

. (MeV)	ln[S-factor(MeV-b)] (Best Fitting) 4.04%	ln[S-factor(MeV-b)] (Formula)	ln[S-factor(MeV-b)] (Best Fitting) 4.215%	ln[S-factor(MeV-b)] (Formula)	ln[S-factor(MeV-b)] (Best Fitting) 3.801%	ln[S-factor(MeV-b)] (Formula)	ln[S-factor(MeV-b)] (Best Fitting) 3.028%	ln[S-factor(MeV-b)] (Formula)	ln[S-factor(MeV-b)] (Best Fitting) 1.974%	ln[S-factor(MeV-b)] (Formula)
5.5	31.055±1.255	31.301	32.461±1.368	32.684	33.534±1.275	33.897	35.503±1.075	35.819	36.214±0.715	37.067
5.75	30.873±1.247	31.182	32.466±1.368	32.584	33.375±1.269	33.821	35.422±1.073	35.803	36.473±0.720	37.128
6	30.679±1.239	31.048	32.325±1.363	32.467	33.216±1.263	33.727	35.319±1.069	35.769	36.645±0.723	37.174
6.25	30.473±1.231	30.899	32.087±1.352	32.334	33.054±1.256	33.615	35.193±1.066	35.716	36.737±0.725	37.203
6.5	30.255±1.222	30.735	31.791±1.340	32.183	32.886±1.250	33.484	35.046±1.061	35.645	36.758±0.726	37.217
6.75	30.026±1.213	30.556	31.472±1.327	32.016	32.712±1.243	33.336	34.881±1.056	35.556	36.715±0.725	37.216
7	29.787±1.203	30.362	31.154±1.313	31.832	32.527±1.236	33.170	34.698±1.051	35.449	36.618±0.723	37.198
7.25	29.539±1.193	30.152	30.853±1.300	31.631	32.329±1.229	32.986	34.498±1.045	35.324	36.474±0.720	37.165
7.5	29.281±1.183	29.928	30.577±1.289	31.413	32.117±1.221	32.784	34.284±1.038	35.180	36.292±0.716	37.117
7.75	29.015±1.172	29.689	30.328±1.278	31.179	31.887±1.212	32.564	34.056±1.031	35.019	36.079±0.712	37.052
8	28.741±1.161	29.434	30.096±1.269	30.928	31.637±1.203	32.326	33.816±1.024	34.839	35.844±0.708	36.972
8.25	28.459±1.150	29.165	29.865±1.259	30.659	31.365±1.192	32.070	33.566±1.016	34.641	35.595±0.703	36.876
8.5	28.171±1.138	28.880	29.613±1.248	30.374	31.068±1.181	31.796	33.306±1.009	34.424	35.340±0.698	36.764
8.75	27.877±1.126	28.581	29.306±1.235	30.072	30.744±1.169	31.504	33.039±1.000	34.190	35.088±0.693	36.637
9	27.577±1.114	28.266	28.903±1.218	29.754	30.391±1.155	31.195	32.766±0.992	33.937	34.846±0.688	36.494
9.25	27.272±1.102	27.937	28.357±1.195	29.418	30.005±1.140	30.867	32.488±0.984	33.666	34.622±0.683	36.335
9.5	26.963±1.089	27.592	27.611±1.164	29.066	29.584±1.124	30.521	32.206±0.975	33.377	34.426±0.680	36.161
9.75	26.649±1.077	27.232	26.600±1.121	28.696	29.127±1.107	30.157	31.923±0.967	33.070	34.264±0.676	35.971
10	26.333±1.064	26.858	25.250±1.064	28.310	28.630±1.088	29.776	31.639±0.958	32.745	34.146±0.674	35.765

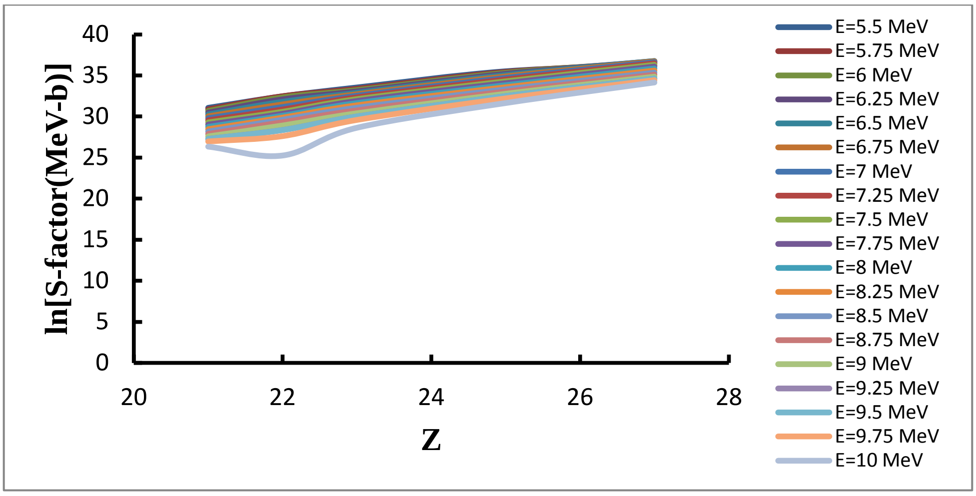
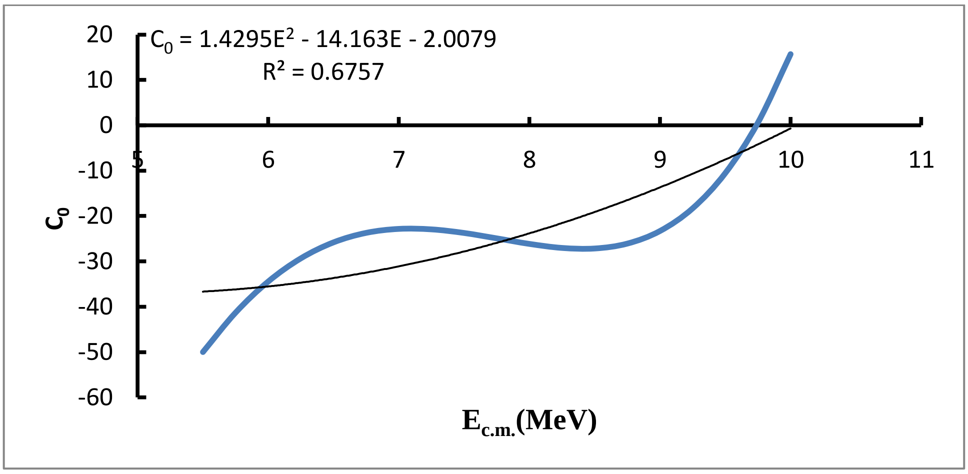
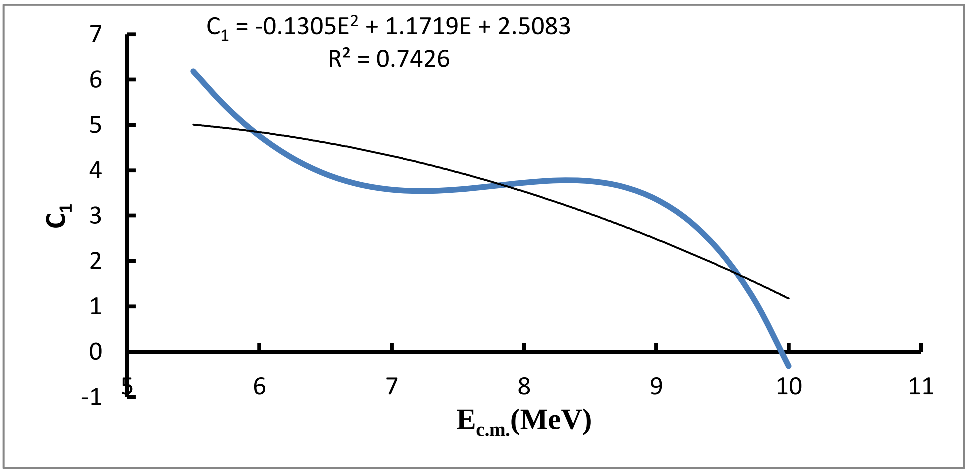


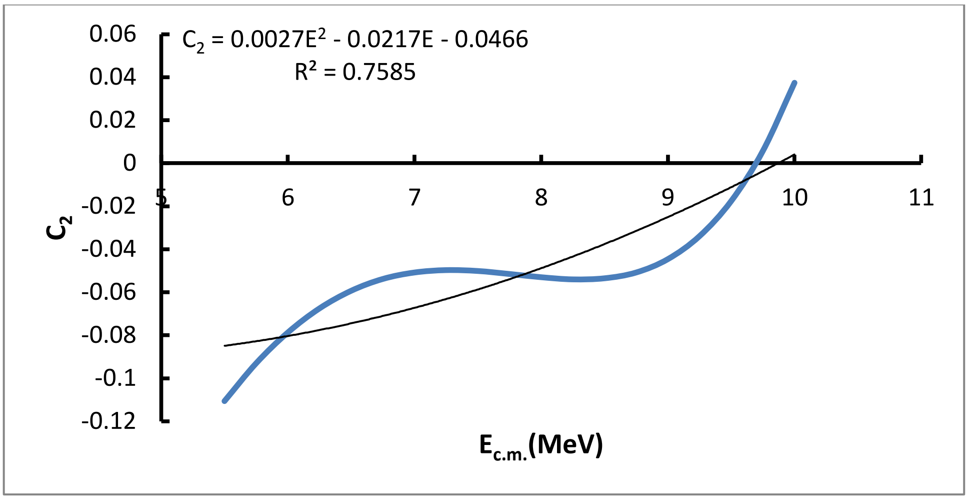
Fig. 1. The variation of the natural logarithm of the astrophysical S-factor S(E) with the atomic number (Z) for the $^{45}Sc(\alpha,n)^{48}V$, $^{48}Ti(\alpha,n)^{51}Cr$, $^{51}V(\alpha,n)^{54}Mn$,$^{55}Mn(\alpha,n)^{58}Co$, and $^{59}Co(\alpha,n)^{62}Cu$ reactions at fixed values of center of mass energies.



(a)



(b)



(c)

Fig. 2. C_i coefficients against the center of mass energy, for C_0 ,C_1, and C_2 respectively. The solid line represents the fitted curve through the data.

4.2. Thermonuclear Reaction Rates Empirical Formulae

The adopted thermonuclear reaction rates N_A<σv> have been utilized to acquire the fitting parameter by utilizing the polynomial expressions (16), (18) and (19) by the steps below:

1. Polynomial expressions were utilized in eq. (19) to fit the computed thermonuclear reaction rates natural logarithm N_A<σv> of the thoughtful medium elements to set the embraced natural logarithm of thermonuclear reaction rates N_A<σv> from the best fitting with a minimum (χ^2) utilizing Eq. (18). The acquired best fitting relationships of the remembered reactions are shown in Eqs. (34, 35, 36, 37, 38, 39, 40, 41, 42, and 43) for the reactions ^{45}Sc(α,n)^{48}V , ^{48}Ti(α,n)^{51}Cr , ^{51}V(α,n)^{54}Mn , ^{50}Cr(α,n)^{53}Fe , ^{55}Mn(α,n)^{58}Co , ^{54}Fe(α,n)^{57}Ni , ^{59}Co(α,n)^{62}Cu , ^{62}Ni(α,n)^{65}Zn , ^{63}Cu(α,n)^{66}Ga , and ^{66}Zn(α,n)^{69}Ge respectively.

^{45}Sc(α,n)^{48}V $x^2 = 1.064$

$$\ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = 0.0018T^5 - 0.0636T^4 + 0.9094T^3 - 6.7134T^2 + 27.463T - 42.483 \quad (34)$$

^{48}Ti(α,n)^{51}Cr x^2=1.216

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = -0.0156T^4 + 0.4584T^3 - 5.103T^2 + 26.92T - 48.328 \quad (35)$$

^{51}V(α,n)^{54}Mn x^2=0.979

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = -0.01T^4 + 0.299T^3 - 3.4636T^2 + 19.663T - 37.011 \quad (36)$$

^{50}Cr(α,n)^{53}Fe x^2=0.445

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = 0.0281T^3 - 0.8253T^2 + 8.9513T - 24.344 \quad (37)$$

^{55}Mn(α,n)^{58}Co x^2=0.318

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = -0.0053T^4 + 0.1798T^3 - 2.4157T^2 + 16.122T - 34.73 \quad (38)$$

^{54}Fe(α,n)^{57}Ni x^2=0.57

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = -0.0093T^4 + 0.2984T^3 - 3.7055T^2 + 22.31T - 47.654 \quad (39)$$

^{59}Co(α,n)^{62}Cu x^2=0.849

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = 0.0013T^5 - 0.0504T^4 + 0.7943T^3 - 6.5501T^2 + 30.053T - 55.065 \quad (40)$$

^{62}Ni(α,n)^{65}Zn x^2=0.597

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = -0.0103T^4 + 0.3309T^3 - 4.1158T^2 + 24.749T - 53.036 \quad (41)$$

^{63}Cu(α,n)^{66}Ga $x^2 = 1.475$

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = 0.0048T^5 - 0.1684T^4 + 2.3605T^3 - 16.76T^2 + 63.115T - 100.95 \quad (42)$$

^{66}Zn(α,n)^{69}Ge $x^2 = 0.422$

$$ln[N_A\langle\sigma v\rangle(cm^3\ s^{-1}\ mol^{-1})] = 0.0048T^5 - 0.1689T^4 + 2.3715T^3 - 16.888T^2 + 63.973T - 102.72 \quad (43)$$

2. At fixed values of T_9, the variation of the natural logarithm of the thermonuclear reaction rates with the physical parameter atomic number Z has been fitted to the polynomial expression utilizing Eq. (19). The acquired outcomes are contemplated to set the coefficients of polynomials (C_i).

3. The coefficients of polynomials Ci~~,~~ are plotted versus each value of T_9 and fitted to satisfactory

the polynomial expression were shown in Eq. (16).
4. The last formula of a set of reactions has been determined by utilizing the combination of the two polynomials to show the systematic manner of the reactions, which is shown in Eq. (17). The Y Variable is the thermonuclear reaction rates.

4.2.1. The Empirical Formulae Relating the Thermonuclear Reaction Rates to T_9 and the Atomic Number Z of the Target Nucleus

The empirical formulae relating to the thermonuclear reaction rates N_A<σv> ($cm^3 s^{-1} mol^{-1}$) with both T_9 and Z were performed as the steps below:

1- At fixed values of the T_9 from 6 to 10 10^9 K in steps of 0.25 10^9 K for the $^{45}Sc(\alpha,n)^{48}V$, $^{48}Ti(\alpha,n)^{51}Cr$, $^{51}V(\alpha,n)^{54}Mn$, $^{55}Mn(\alpha,n)^{58}Co$, $^{62}Ni(\alpha,n)^{65}Zn$, and $^{66}Zn(\alpha,n)^{69}Ge$ reactions, the natural logarithm of the thermonuclear reaction rates will vary with the atomic number Z this shown in Fig. (3). The data fitted to the polynomial expression as the same as Eq. (30), Where $Y = \ln[N_A\langle\sigma v\rangle]$, X=Z, with free parameters C_i (C_0, C_1, and C_2).

2- The adopted thermonuclear reaction rates have been used as a function of Z at fixed T_9 utilizing the computer program Excel to acquiring the fitting expressions and then used to calculate the fitting parameters. The obtained results are presented in Table (5).

3- The obtained free parameters C_i (C_0, C_1, and C_2), as presented in Table (5) are plotted versus with the fixed values of T_9 from 6 to 10 10^9K in steps of 0.25 10^9K as presented in Fig.(4),and then the acquired coefficients of polynomials C_i have been fitted to the polynomial expression:

$$C_i = \sum_{j=0}^{2} C_{ij} T_9{}^j \qquad (44)$$

The combination of the two polynomials Eq. (30) and Eq. (44) takes the shape of the following formula range T_9 from 6 to 10 10^9K in steps of 0.25 10^9K:

$$Y = \sum_{i=0}^{2} \left(\sum_{j=0}^{2} C_{ij} T_9{}^j \right) X^i \qquad (45)$$

Where Y=ln[N_A<σv>], T_9 is the temperature in 10^9K, and X=atomic numberZ

$$Y = \sum_{i=0}^{2} (C_{i0}T_9^0 + C_{i1}T_9^1 + C_{i2}T_9^2) X^i$$

$$Y = C_{00}T_9^0X^0 + C_{01}T_9^1X^0 + C_{02}T_9^2X^0 + C_{10}T_9^0X^1 + C_{11}T_9^1X^1 + C_{12}T_9^2X^1 + C_{20}T_9^0X^2 + C_{21}T_9^1X^2 + C_{22}T_9^2X^2 \qquad (46)$$

Where (C_{00}, C_{01}, $C_{02,}$, C_{10}, C_{11}…. C_{22}) are free parameters and their values are shown in the matrix below:

$$\begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 42.691 & -16.998 & 1.7602 \\ -3.0522 & 1.4998 & -0.145 \\ 0.0375 & -0.0265 & 0.0027 \end{bmatrix}, \begin{bmatrix} R^2 = 0.9717 \\ R^2 = 0.9645 \\ R^2 = 0.9643 \end{bmatrix}$$

The acquired formula of a set of reactions such as $^{45}Sc(\alpha,n)^{48}V$, $^{48}Ti(\alpha,n)^{51}Cr$, $^{51}V(\alpha,n)^{54}Mn$, $^{55}Mn(\alpha,n)^{58}Co$, $^{62}Ni(\alpha,n)^{65}Zn$, and $^{66}Zn(\alpha,n)^{69}Ge$ has been used to calculate the thermonuclear reaction rates $N_A\langle\sigma v\rangle$ for each of the above reactions and compared with the adopted thermonuclear reaction rates calculated from the fitting expressions and shown to be in a good agreement and the comparison of the two results are shown in Table (6).

Table 7 presents the comparison of thermonuclear reaction rates of some (α,n) medium elements reactions with other works as Roughton et.al. (Roughton *et al.*, 1983)

Table 5. Free parameters C_i (C_0, C_1, and C_2) as a function of T_9.

T9 (109 K)	C0	C1	C2
6	1.8494	0.9114	-0.0281
6.25	3.9765	0.7593	-0.0247
6.5	6.3605	0.5865	-0.021
6.75	8.9003	0.4013	-0.017
7	11.488	0.2126	-0.013

7.25	14.021	0.0288	-0.0091
7.5	16.418	-0.1433	-0.0056
7.75	18.627	-0.2998	-0.0023
8	20.644	-0.4404	0.0006
8.25	22.523	-0.5696	0.0032
8.5	24.388	-0.6979	0.0058
8.75	26.451	-0.8427	0.0086
9	29.02	-1.0295	0.0124
9.25	32.512	-1.2928	0.0176
9.5	37.472	-1.6774	0.0253
9.75	44.58	-2.2392	0.0366
10	54.667	-3.0466	0.0528

Comparison between polynomial fitting expression (Best Fitting) of the adopted astrophysical S-Factor of (α,n) medium element reactions with those computed from Eq. (46).

T9 (109 K)	^{45}Sc(α,n)^{48}V		^{48}Ti(α,n)^{51}Cr		^{51}V(α,n)^{54}Mn		^{55}Mn(α,n)^{58}Co		^{62}Ni(α,n)^{65}Zn		^{66}Zn(α,n)^{69}Ge	
	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 4.04%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 4.215%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 3.801%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 3.028%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 3.785%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)	ln[Na<σv> (cm3 s-1 mol-1)] (Best Fitting) 9.071%	ln[Na<σv> (cm3 s-1 mol-1)] (Formula)
6	8.614±0.348	8.613	8.281±0.349	8.294	7.901±0.300	7.927	7.005±0.212	7.048	5.415±0.205	5.364	3.824±0.347	3.998
6.25	9.060±0.366	9.027	8.696±0.367	8.710	8.325±0.316	8.348	7.479±0.226	7.488	5.941±0.225	5.858	4.459±0.404	4.545
6.5	9.485±0.383	9.430	9.091±0.383	9.111	8.724±0.332	8.751	7.916±0.240	7.907	6.427±0.243	6.330	5.057±0.459	5.072
6.75	9.889±0.400	9.821	9.471±0.399	9.497	9.101±0.346	9.136	8.323±0.252	8.303	6.879±0.260	6.779	5.620±0.510	5.580
7	10.275±0.415	10.201	9.841±0.415	9.867	9.461±0.360	9.502	8.701±0.263	8.677	7.301±0.276	7.205	6.148±0.558	6.067
7.25	10.643±0.430	10.568	10.202±0.430	10.222	9.804±0.373	9.850	9.054±0.274	9.029	7.699±0.291	7.608	6.642±0.602	6.534
7.5	10.994±0.444	10.924	10.556±0.445	10.561	10.134±0.385	10.179	9.385±0.284	9.359	8.076±0.306	7.989	7.100±0.644	6.981
7.75	11.331±0.458	11.268	10.904±0.460	10.885	10.450±0.397	10.490	9.697±0.294	9.667	8.435±0.319	8.346	7.523±0.682	7.409
8	11.653±0.471	11.600	11.243±0.474	11.193	10.751±0.409	10.783	9.990±0.302	9.952	8.777±0.332	8.681	7.912±0.718	7.816
8.25	11.964±0.483	11.921	11.571±0.488	11.487	11.036±0.419	11.058	10.266±0.311	10.216	9.102±0.345	8.992	8.269±0.750	8.203
8.5	12.266±0.496	12.230	11.882±0.501	11.764	11.302±0.430	11.314	10.526±0.319	10.457	9.411±0.356	9.281	8.600±0.780	8.570
8.75	12.563±0.508	12.527	12.171±0.513	12.027	11.547±0.439	11.552	10.770±0.326	10.676	9.702±0.367	9.547	8.911±0.808	8.918
9	12.860±0.520	12.812	12.431±0.524	12.274	11.765±0.447	11.772	10.997±0.333	10.873	9.973±0.377	9.790	9.215±0.836	9.245
9.25	13.164±0.532	13.085	12.652±0.533	12.506	11.953±0.454	11.973	11.207±0.339	11.047	10.220±0.387	10.010	9.525±0.864	9.553
9.5	13.483±0.545	13.347	12.824±0.541	12.722	12.102±0.460	12.156	11.399±0.345	11.200	10.440±0.395	10.207	9.860±0.894	9.840
9.75	13.826±0.559	13.597	12.935±0.545	12.923	12.207±0.464	12.321	11.571±0.350	11.330	10.626±0.402	10.382	10.245±0.929	10.108
10	14.207±0.574	13.835	12.972±0.547	13.109	12.259±0.466	12.467	11.720±0.355	11.439	10.774±0.408	10.533	10.710±0.972	10.355

Table 7. comparison of the thermonuclear reaction rates in natural logarithm ln[Na<σv> (cm^3 s^{-1} mol^{-1})] of some (α,n) medium element reactions with other works.

T9 (109 K)	^{48}Ti(α,n)^{51}Cr		^{51}V(α,n)^{54}Mn		^{50}Cr(α,n)^{53}Fe		^{55}Mn(α,n)^{58}Co	
	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work
2	-10.054	-11.904	-10.680	-9.599	-15.936	-16.080	-12.652	-13.626

3	-1.347	-2.179	-2.120	-1.796	-5.655	-5.707	-3.507	-3.953
4	3.401	2.918	2.708	2.723	-0.211	-0.181	1.629	1.283
5	6.446	6.098	5.914	5.780	3.258	3.313	5.011	4.654
6	8.556	8.290	8.189	7.973	5.670	5.728	7.378	7.015
7	10.127	9.894	9.903	9.607	7.496	7.490	9.116	8.755
8	11.327	11.117	11.184	10.860	8.882	8.826	10.491	10.081
9	12.301	12.076	12.206	11.845	9.999	9.869	11.608	11.120
10	13.102	12.846	13.060	12.636	10.897	10.700	12.506	11.952
T9 (109 K)	$^{54}Fe(\alpha,n)^{57}Ni$		$^{59}Co(\alpha,n)^{62}Cu$		$^{63}Cu(\alpha,n)^{66}Ga$		$^{66}Zn(\alpha,n)^{69}Ge$	
	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work	Roughton et al. 1983	Present Work
2	-18.526	-18.474	-16.811	-17.051	-25.945	-25.653	-25.759	-26.138
3	-6.908	-6.867	-5.991	-6.195	-10.871	-11.147	-10.820	-11.238
4	-0.968	-0.942	-0.174	-0.380	-3.270	-3.901	-3.244	-3.708
5	2.708	2.699	3.526	3.311	1.335	0.427	1.386	0.826
6	5.193	5.179	6.131	5.865	4.431	3.291	4.522	3.841
7	7.003	6.978	8.039	7.726	6.659	5.317	6.791	5.979
8	8.434	8.339	9.547	9.134	8.343	6.821	8.517	7.568
9	9.547	9.401	10.714	10.231	9.680	7.975	9.852	8.789
10	10.519	10.248	11.695	11.104	10.736	8.887	10.951	9.753

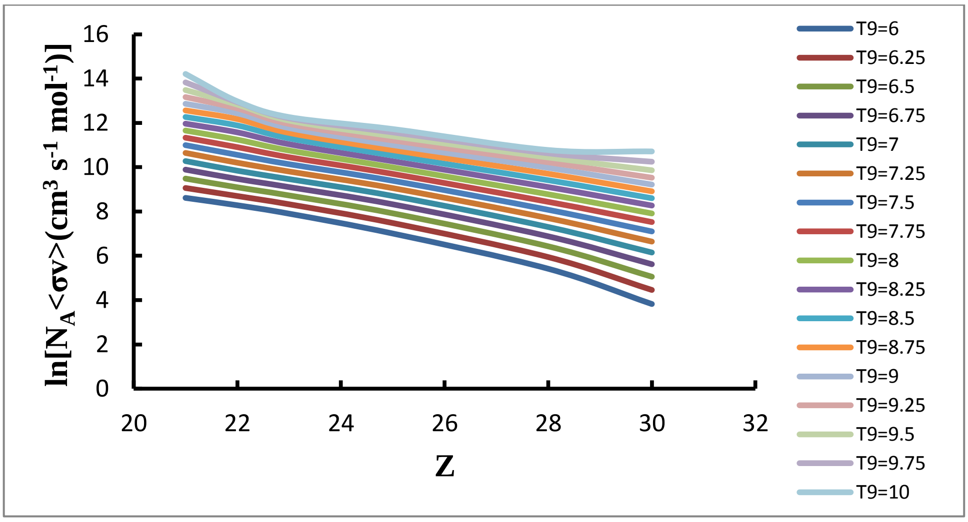
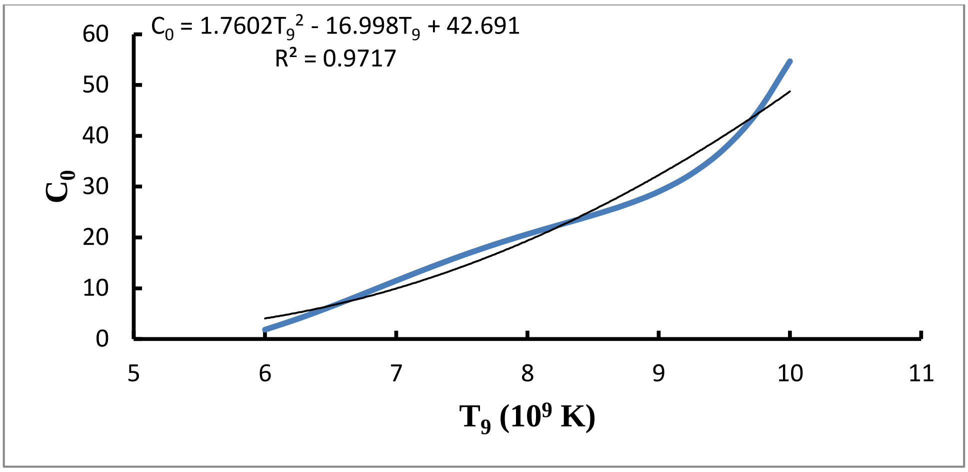
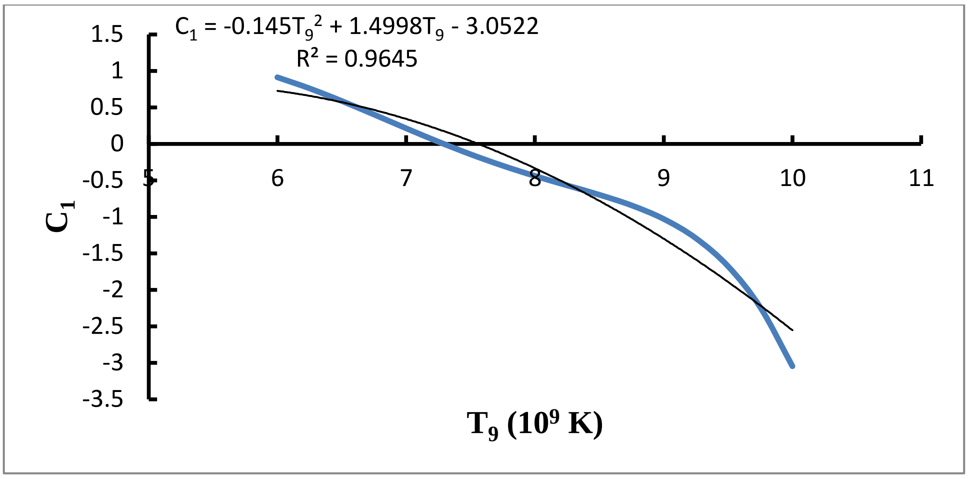


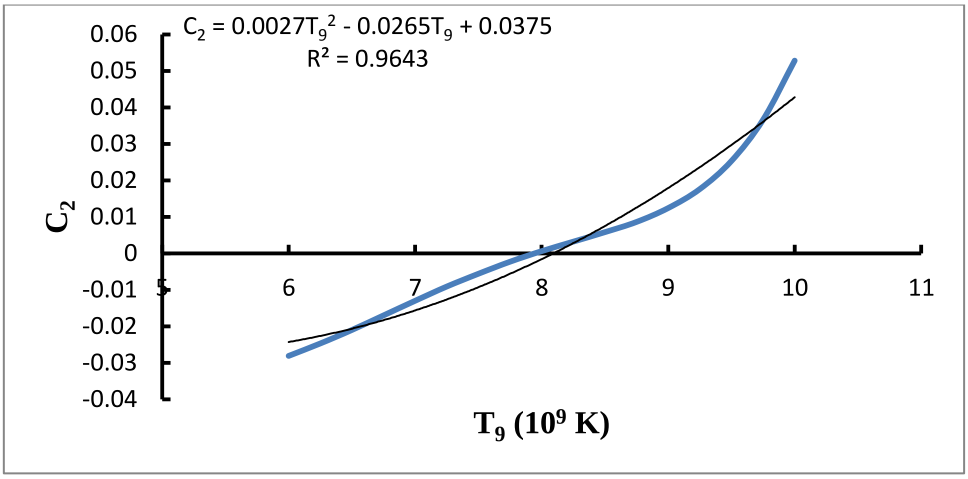
Fig. 3. The variation of the natural logarithm of the thermonuclear reaction rates with the atomic number Z for the $^{45}Sc(\alpha,n)^{48}V$, $^{48}Ti(\alpha,n)^{51}Cr$, $^{51}V(\alpha,n)^{54}Mn$, $^{55}Mn(\alpha,n)^{58}Co$, $^{62}Ni(\alpha,n)^{65}Zn$, and $^{66}Zn(\alpha,n)^{69}Ge$ reactions at fixed values of T_9.



(a)



(b)



(c)

Fig. 4. C_i coefficients against T_9, for C_0, C_1, and C_2 respectively. The solid line represents the fitted curve through the data.

5. Conclusions

1-The astrophysical S-factor, S(E), was starting with an increase and then decreased irregularly by increasing the center of mass energy, this because of Coulomb barrier penetration exp(2πη).
2-The astrophysical S-factor increased with increasing atomic number Z of target nuclei at a fixed center of mass energy.
3-The thermonuclear reaction rates, $N_A<\sigma v>$, were increased with increasing T_9 because by increasing the T_9 the charged interacting particles need to overcome the existing Coulomb barrier.
4-The thermonuclear reaction rates decreased with increasing atomic number Z of target nuclei at fixed T_9 because as Z increased Coulomb barrier increased.
5-The astrophysical S-factor and Thermonuclear reaction rates calculated in the present work are in good agreement with those measured previously by other works.

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