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RESEARCH PAPER

Comparison of the water movement by Richard and Darcy

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A B S T R A C T:

Simulations of water movement in the soil are pervasive, Darcy's law, Darcy- Buckingham's law and Richard's equation are all approved equations for this purpose. The Richard's equation is the most reliable model which is introduced by Lewis Richardson who was the first one to propose that Darcy's law was originally fabricated for flow in the saturated zone while for flow in vadose zone Darcy-Buckingham's law can be used in case of steady state condition. Since it is impossible in nature for flow to be steady, Richard proposed a highly nonlinear partial differential equation (PDE) equation which a combination of Darcy's law and continuity equation. Hydraulic conductivity $K(LT^{-1})$ and moisture content θ ($L^{3}L^{3}$) are two important variables of Richard's equation. Considering the nonlinearity of its variables Richard's equation lacks a general closed form solution. For this purpose, many researchers derived numerical and analytical models to solve Richard's equation. In this paper, Green Ampt Infiltration Equation is presented to solve Richard's equation which is one of the most widely used analytical methods.

KEY WORDS: Darcy law; Darcy- Buckingham law; Richard equation; Vadose zone; Green Ampt method.
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1.INTRODUCTION :

The vadose zone is commonly called the unsaturated zone, the word vadose is derived from a Latin word vadosus, meaning shallow. Which is located between the land surface and the groundwater table. The Vadose zone is a three-phase system contains soil, air, and water (Ballestero et al., 2005).

Vadose zone is of great importance in understanding the groundwater condition, contamination transport, biological degradation. As it acts as a buffer zone between the land surface and aquifers by removing undesirable materials before reaching to aquifers (Selker et al., 1999).

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Henry Darcy (1803-1858) a French scientist who was impressed by flow through a porous medium (Brown, 2002). Darcy designed a special experiment to quantum the flow rate comparing to other physical parameters. His finding the linear relation between pressure differences and the flow rate is of great importance for different scientific disciplines related to porous media. Darcy's law is recognized as the basic law for describing fluid flow in the saturated zone (Kovalchuk and Hadjistassou, 2019).

Edgar Buckingham [1907] was able to make Darcy's law applicable for the vadose zone by making some modifications on the law, where the maximum value at saturation θ_s is higher than actual water content in the medium θ. The fundamental hypothesis is that the saturated hydraulic conductivity Ks, can be replaced by a function of capillary pressure Ψor water content θ as the characteristic of the unsaturated porous medium. Buckingham's work comprises an achievement in the historical background of soil science and the movement of fluid in porous media ((Liu), (Narasimhan, 2007)).

There are two types of flow in nature steady state condition and transient state condition. In the steady state condition, Buckingham law is used where, the influx and outflux rate are equal while for transient state the influx and outflux are not equal in case of heavy rainfall large amount of water enters the soil, some of the water starts to evaporate lessening the amount of the original water by this the influx and outflux rates are different (Hou et al., 2019). Addressing the water in the column and also addressing the precipitation and the evaporation. To describe this, The term of Continuity equation is used that is the derivative of the volumetric water content in respect to time equals the derivative of the flow with respect to space $\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}$ meaning that whatever changes in time need to be changed in space (Kuntz and Grathwohl, 2009). To describe flow in transient state condition under the action of capillary and gravity Richard combined aspects of Darcy's law and mass continuity law (Pinder and Celia, 2006), Richard obtained a nonlinear partial differential equation (PDE), defining the water flow both in the saturated and unsaturated porous media (Navon, 2009).

Due to the complicated relationship between hydraulic conductivity and infiltration parameters Richard's equation becomes nonlinear and difficult to be solved. Many researchers have developed numerical models ((Pour et al., 2011), (Farthing and Ogden, 2017), (Chávez-Negrete et al., 2018), (Oulhaj et al., 2018)) and analytical models ((Yuan and Lu, 2005), (Tracy, 2006), (Menziani et al., 2007), (Tracy, 2007) (Zhang et al., 2016)) to solve Richard's equation. One of the analytical methods that is used in this paper is Green Ampt Infiltration Equation. This equation is widely used in hydrologic models for *it is acceptable physical basis and it is accurate results.*

Energy state (matric potential and gravitational potential) has a basic role in the distribution of water with depth at a given time. If there is no flow, it is obvious that the gradient of total potential is zero. Figure 1 (a) shows the hydrostatic profile for the case where the water table is present. If water flows vertically downward at a steady rate in a homogeneous medium, the total gradient must be constant, but the matric pressure does not cancel out the gravitational potential, as illustrated in Figure 1 (b). While if water flows vertically upward such as evaporation the condition becomes drier and the hydraulic potential becomes negative like the example in Figure 1 (c) (Hillel, 2012).

STUDY AREA:

The study was carried out on a yard at College of Engineering, Salahaddin University as shown in figure (2). The geographical coordinates of the research site are 36.1632°N and 44.0162°E. The laboratory tests were carried out in the soil laboratory of the civil engineering department at the College of Engineering Salahaddin University-Erbil. The soil samples were analyzed for particle size distribution by the hydrometer method. Soil samples were collected by excavating to a depth of 1.10 m. pickaxe was utilized in the digging process. Samples were collected at three depths 50, 80 and 110 cm. samples were carried in Ziploc bags and labeled. Soil texture classes are shown in table 2.

The soil texture at the 50 cm depth is classified as silty clay according to USDA (United States Department of Agriculture) soil texture identification triangle, the soil in the second layer at 80 cm depth is classified as clay while the soil at the third layer at 110 cm is classified as silty clay loam.

2. MATERIALS AND METHODS

Three equations have been discussed here Darcy law, Darcy- Buckingham law and Richard's equation.

2.1 Darcy's law

Darcy's law is applicable to the saturated zone where there are only two phases water and soil (Simmons, 2008). Darcy interpreted his experimental data from a vertical column of sand with an equation of the form:

$$q = -k\frac{\Delta h}{L} \qquad \dots\dots\dots \qquad (1)$$

Where, q is the flux (discharge per unit area, with units of length per time [LT^{-1}]), K is hydraulic conductivity at saturation [LT^{-1}] and $\frac{\Delta h}{L}$ is hydraulic gradient. The negative sign in Darcy's law is due to the flow of fluid from high pressure to low pressure.

2.2. Darcy- Buckingham extension law

The Darcy-Buckingham equation is adequate for describing unsaturated flow only if the soil water content is not changing in time ($\frac{\partial \theta}{\partial t} = 0$). In nature, this is difficult. When θ and q alter in time, must combine Eq. (1) with the equation of continuity. The equation of continuity relates the time rate of change of θ to the spatial rate of change of q, $\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}$. The resulting differential equation is strongly non-linear and its solution even for simple conditions is most difficult. Generally, equation (1) itself is not satisfactory for the solution of such hydrologically important processes as evaporation, infiltration, drainage, subsurface flow (Kutilek et al., 2007). Darcy-Buckingham is only valid for steady-state condition as shown in equation (2):

$$q = -k(\Psi m)\left[\frac{\partial \Psi m}{\partial z} + \frac{\partial \Psi z}{\partial z}\right] \quad \text{........} \quad (2)$$

Since $\frac{\partial \Psi z}{\partial z} = 1$, so Eq. (2) becomes:

$$q = -k(\Psi m)\left[\frac{\partial \Psi m}{\partial z} + 1\right] \quad \dots\dots\dots \quad (3)$$

Where, z is vertical axis (L) and Ψm is matric potential function (L).

Matric pressure and water content in the vadose zone have an impact on the porous media and in turn, are affected by the conditions in the saturated zone and land surface. (Nimmo et al., 2005)

2.3Richard's equation

Richard's equation, the partial differential equation (PDE) that describes the fluid flow in the variably saturated porous media is accomplished by merging Darcy's law with continuity equation (Nimmo, 2006). Richard's equation is written as:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\Psi)\left(\frac{\partial \Psi}{\partial z} + 1\right)\right) \dots\dots\dots\dots \quad (4)$$

Where θ is volumetric water content [L^3/L^3], t is time (T), z is the elevation (L), Ψ is pressure head (L) and K is hydraulic conductivity (LT^{-1}).

Green Ampt Infiltration Equation simulates infiltration rate by the following equation (Rawls et al., 1983):

$$f = k\left[\frac{|\Psi|\Delta\theta + F}{F}\right] \quad \dots\dots\dots\dots \quad (5)$$

where k is hydraulic conductivity (cm h^{-1}), f is infiltration rate (cm h^{-1}), F is cumulative infiltration (cm), Ψ *is suction head (cm) and* Δθ is the difference between initial water content and saturated water content ($cm^3\ cm^{-3}$).

Darcy's law simulates water flows into the soil by equation (1). The q in equation (1) is similar to the infiltration rate (f) of Green Ampt method with opposite sign.

$$f = K(h)\frac{\partial H}{\partial z} \quad \dots\dots\dots\dots \quad (6)$$

$$F(t) - |\Psi|\Delta\theta \ln\left(\left|1 + \frac{F(t)}{|\Psi|\Delta\theta}\right|\right) = K(t) \quad \dots(7)$$

$$\Delta\theta = \theta_e(1 - \sigma_e) \quad \dots\dots\dots\dots \quad (8)$$

Where θ_e is Porosity and σ_e is Effective saturation.

3. RESULTS:

The value of parameters of Green Ampt Equation are obtained from the table of (Green and Ampt Parameters According to Soil Texture Classes (Rawls et al., 1983)), shown in table 1.

The soil at 0-50 cm is silty clay the parameters are obtained from table 1. Infiltration rate is computed as follows:

Assume $\sigma_e = 0.30$

therefore,

Δθ = 0.424 (1- 0.30) = 0.297

$$F(t) - |30.66| * \Delta\theta * \ln\left(\left|1 + \frac{F(t)}{|30.66| * \Delta\theta}\right|\right) = 0.04\ (t)$$

At time = 0.2 hour

F= -1.88

$$f = k\left[\frac{|\Psi|\Delta\theta + F}{F}\right]$$

f = *-0.153 cm/hr, q= 0.153cm/hr.*

The soil at 50-80 cm is clay, the parameters are obtained from table1. Infiltration rate is computed as follows:

Assume $\sigma_e = 0.30$
therefore,
$\Delta\theta$ = 0.412 (1- 0.30) = 0.288

$$F(t) - |27.72| * \Delta\theta * \ln\left(\left|1 + \frac{F(t)}{|27.72| * \Delta\theta}\right|\right) = 0.03\ (t)$$

At time = 0.2 hour
F= 1.56

$$f = k\left[\frac{|\Psi|\Delta\theta + F}{F}\right]$$

f = *0.18 cm/hr, q= 0.18 cm/hr.*

The soil at 80-110 cm is silty clay loam, the parameters are obtained from table 1. Infiltration rate is computed as follows:
Assume $\sigma_e = 0.30$
therefore,
$\Delta\theta$ = 0.451 (1- 0.30) = 0.316

$$F(t) - |21.54| * \Delta\theta * \ln\left(\left|1 + \frac{F(t)}{|21.54| * \Delta\theta}\right|\right) = 0.06\ (t)$$

At time = 0.2 hour
F= 1.99

$$f = k\left[\frac{|\Psi|\Delta\theta + F}{F}\right]$$

f = *0.26 cm/hr, q= 0.26 cm/hr.*

4. DISCUSSION:

This paper is intended to show the difference between Darcy law and Richard's equation on the state of the science for calculating the flow of water in the unsaturated zone. The focus is mostly on the vadose since it's a critical zone and it is considered as a controlling agent for most of the groundwater cases.

Darcy's law is an empirical equation that has been developed for the saturated zone. Where all the pores are filled with water. While it is not applicable for the unsaturated zone, where the pore spaces are not fully saturated with water so the remaining portions are filled with air. Physical properties of saturated and unsaturated zone play a great role in the applicability of the equations. In unsaturated zone, two factors actuate water flux, pressure head (Ψ) and properties of the medium (hydraulic conductivity (K)). Because of high the dependency of K on Ψ it is not possible to directly solve Richard's equation.

5. CONCLUSION:

The governing equation for flow in saturated media is Darcy's law. While for unsaturated media Richard's equation is considered as the governing equation. The interaction of multiple phases of vadose zone complicates the dynamics of vadose zone water. Vadose zone phenomena are tremendously reactive to the proportions of the phases. Richard's equation is a complicated nonlinear (PDE). In order to solve Richard's equation, there are several methods for simplifying Richard's equation, Analytical solutions as Green Ampt Infiltration Equation, numerical solutions by using the finite difference or finite element methods. Because of this nonlinearity and complications in the unsaturated zone, it is exposed to a greater degree of inaccuracy than most disciplines of physical science There are also software that numerically calculates the water movement such as HYDRUS program.

Conflicts of interest:
We have no conflicts of interest to disclose.

Table 1 (Green and Ampt Parameters According to Soil Texture Classes.)

Soil texture class	Effective porosity (cm^{-3} cm^{-3})	Wetted capillary pressure, **Ψ** (cm)	Hydraulic conductivity, K(cm h^{-1}).
Silty clay	0.424	30.66	0.04
Clay	0.412	27.72	0.03
Silty clay loam	0.451	21.54	0.06

Table 2 (Physical properties of the soil at the three layers.)

Depth (cm)	Sand (%)	Silt(%)	Clay (%)	Texture class
0-50	8.63	46.27	44.73	Silty clay
50-80	6.14	28.88	64.44	Clay
80-110	4.36	63.40	32.07	Silty clay loam

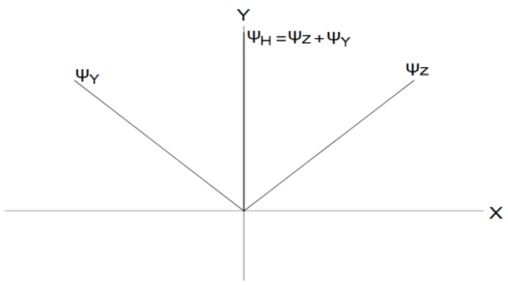

Figure 1(a) Profile of gravitational, matric potential and total potential at static water.

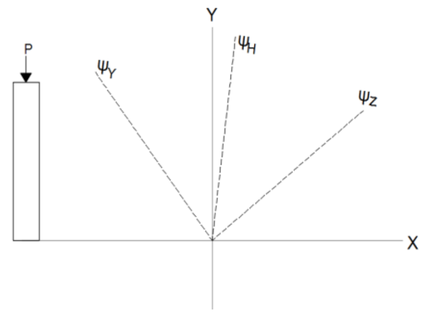

Figure 1 (b) Profile of gravitational, matric potential and total potential at steady downward (precipitation)

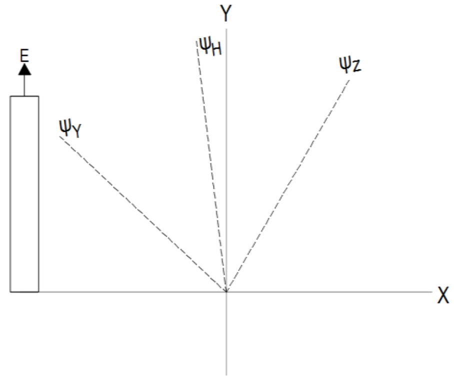

Figure 1 (c) Profile of gravitational, matric potential and total potential at unsteady flow.

Ψ_Y = Matric potential

Ψ_z = *Gravitational potential*

Ψ_H = Total potential

P= precipitation

E= Evaporation

Figure 2 The location of the study area.

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