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RESEARCH PAPER

Comparative Additional Factor for Diffraction Loss On (TEM01) Mode from That of (TEM00) Mode

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A B S T R A C T:

In this paper, the tendency is towards looking for a correction factor in the diffraction loss from (TEM_{01}) in order to search for the same parameters, diffraction loss in a cavity has standing (TEM_{01}) modes. This will be through a lengthy analysis in the far-field intensities the waves via – Gaussian beam shaped wave patterns. The analysis is in the same criteria as that of (TEM_{00}) modes, except for a little complicating factor in the round-trip by pass on the surface of the aperture mirror. The results showed that the diffraction loss procedure is just the continuate for the earlier work, except that the field intensities change for TEM_{01}, as that of TEM_{00}, as such, it has a direct effect on the equation fitting it. The Combination of experimental and Theoretical results in this study are in excellent agreement as in g = 0.80 and they were both functions of the physical parameters of the active medium and the laser resonator.

KEY WORDS: Gas Laser, Symmetric Resonator, Diffraction Loss.
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1. INTRODUCTION:

Since the end of sixties of the last century, few tendencies were done for manipulating and formation of mathematical equation for the diffraction loss parameter in (TEM_{00}) modes (Bhawal, 2005). This is a linear criterion mostly achieved by Li (Kogelnik and Li, 1966) and co-workers. The approach is done through a modelling for the Gaussian beam propagated along the principal axis of the cavity resonator and considering the finiteness of the mirror dimensions and the resonator length (Siegman, 1986), as shown in figure (**1**).

This is done through defining the loss factor from the parallel component of the electric field in the wave pattern. Many parameters have in relationship with this process and all of them are describing the geometrical and physical properties of a typical cavity in those laser tubes available around? The far field wave patterns have almost serval transverse results and phase reversals, with either even or odd symmetry in simple cases

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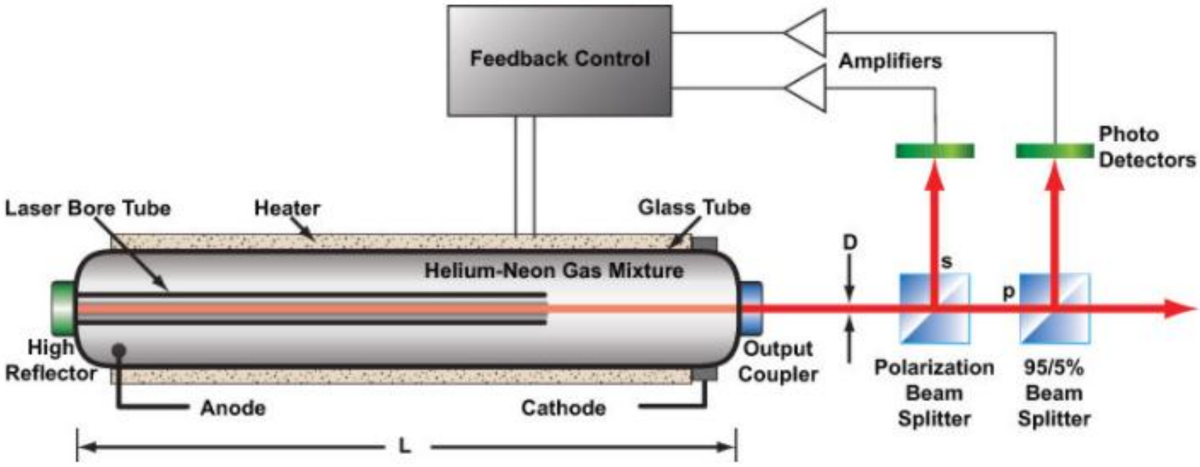

Figure (1): Schematic diagram of a helium-neon laser consisting the optical cavity as comprised with the active medium and two parallel facing mirrors.

These transverse modes spread inside the cavity are generally larger than cavity length, which make the diffraction losses higher than those of lower term transverse modes (Verdeyen, 1995).

In typical gas lasers, the resonators have finite lengths, ranging from few centimeters to 50 cm, so that the radius of curvature for the spherical-identical mirrors is twice to three times larger than this length (Hu et al., 2020). Also, the out-put coupler has an aperture such that the diameter is larger than the beam waist about twice its diameter to reduce losses on the periphery, these restrictions are on the behalf of the finiteness for Fresnel number, N (5-10) as optimum.
Traditionally, the diffraction loss in Resonators is a phenomenon that occurs naturally since a very few percent of the incident radiation spreads out on the periphery of the output coupler (M_2) and it is depending on the geometrical parameters of what is called the resonator (aperture). This means that these optimizing parameter hold a complicated variation in their normal values and individual relationships.

2. Theory of Diffraction Loss (TEM_{01}) in Symmetric Resonator

The Diffraction loss, generally is the fractional loos due to the power captured by the output coupler and the total power incident on it. Thus; if δ_{mn} is the loss due to diffraction, then lose is given by (Svelto and Hanna, 1998):

$$\delta_{mn} = 1 - \frac{P_{transmitted}}{P_{incident}} \,..(1A)$$

Since the symmetric Gaussian beam spreads out from both sides of the minimum beam waist (ω_0), then the losses are due to absorption, scattering and diffraction on the surface of the out-put mirror are possible even in typical and well-designed resonators. When the beam arrives (M_2), then the only considerable part of loss is due to diffraction.
This loss is apparently defined as the fractional value for the attenuations between the incident power and the transmitted power from (M_2). Hence it is given by (Zhu et al., 2021):

$$\delta_{mn} = 1 - |\sigma_{mn}|^2 \,..(1B)$$

The calculated diffraction loss of m, n mode from the ratio of the energy captured by the mirror to the total energy in the $E - field$ at the mirror is given by (Svelto and Hanna, 1998);

$$\delta_{mn} = 1 - \frac{\iint_{\Omega} |E_{mn}(x,y,z)|^2 dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E_{mn}(x,y,z)|^2 dxdy} \,..(1C)$$

where Ω is the aperture representing the mirror, $E_{mn}(x,y,z)$ is the electric field at (m,n) mode is given by (Verdeyen, 1995):

$$E_{mn}(x,y,z) = E_0 \frac{\omega_0}{\omega_{(z)}} H_m\left(\frac{\sqrt{2}x}{\omega}\right) H_n\left(\frac{\sqrt{2}y}{\omega}\right) e^{\frac{-(x^2+y^2)}{\omega^2(z)}} e^{\frac{-j\pi(x^2+y^2)}{\lambda R(z)}} e^{j(1+m+n)tan^{-1}\left(\frac{\lambda z}{\pi\omega_0^2}\right)} \,..(2)$$

where E_0 is the electric field amplitude at the origin $(0,0)$, ω_0 is the waist radius is given by this equation (Lakhotia et al., 2020); $\omega_0^2 = \frac{\lambda z_0}{\pi} = ..(3)$, here (z_0) is called Rayleigh range (Karbstein, 2018, Bisti et al., 2021).
also $\omega_{(z)}$ is beam radius is given by this equation (Urunkar et al., 2018, Falcón et al., 2019, Sasnett, 2020); $\omega_{(z)} = \omega_0\sqrt{1+\left(\frac{\lambda z}{\pi\omega_0^2}\right)^2}\,..(4)$,

and $R_{(z)}$ is radius of curvature of the Gaussian beam is given by this equation (Sasnett, 2020, Wang et al., 2021, Sohn et al., 2019);

$$\omega_{(z)} = z\left[1+\left(\frac{\pi\omega_0^2}{\lambda z}\right)^2\right]..(5),$$

Hermite polynomials of order q is given by this equation (Zhou et al., 2018, Ast et al., 2021);

$$H_q(x) = (-1)^q e^{-\frac{x^2}{2}}\frac{d^q}{dx^q}\left(e^{-\frac{x^2}{2}}\right)\;..(6)$$

However, an approximation technique could be generated by using some elementary physical reasoning of TEM_{01} mode, as shown in figure (**2**).

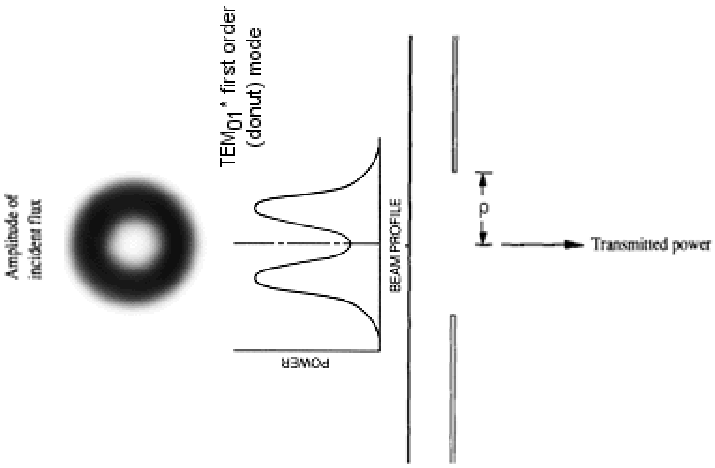

Figure (2): Transmission through an aperture.

The calculated the diffraction loss (δ_{01}) from the ratio of the energy captured by the mirror to the total energy in the $E - field$ at the mirror is given by (Zhu et al., 2020, Borneis et al., 2021, Coldren, 2021):

$$\delta_{01} = 1 - \frac{\iint_{\Omega}|E_{01}(x,y,z)|^2 dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|E_{01}(x,y,z)|^2 dxdy}\;..(7)$$

Each of the numerator and denominator of equation (1C) have been summarized from the Gaussian beam distribution of through Hermite part of the modified function of $E_{01}(x,y,z)$ (Aksenov et al., 2020, Hashemian and Akou, 2020) Thus:

$$E_{01}(x,y,z) = E_0\frac{\omega_0}{\omega_{(z)}}H_0\left(\frac{\sqrt{2}x}{\omega}\right)H_1\left(\frac{\sqrt{2}y}{\omega}\right)e^{\frac{-(x^2+y^2)}{\omega^2(z)}}\,e^{\frac{-j\pi(x^2+y^2)}{\lambda R(z)}}\,e^{j(1+m+n)tan^{-1}\left(\frac{\lambda z}{\pi\omega_0^2}\right)}..(8)$$

Here, in polar coordinates: $x^2+y^2=r^2, x = r\,cos\phi, y = r\,sin\phi$. Also, the zero order Hermite polynomial is $H_0\left(\frac{\sqrt{2}r cos\phi}{\omega}\right) = 1$, and the first order Hermite polynomial is $(H_1\left(\frac{\sqrt{2}r\,sin\phi}{\omega}\right) = \frac{\sqrt{2}r\,sin\phi}{\omega})$ (George et al., 2013), besides, the second and third exponential terms are equal to a unity, after squaring them and taking their complex conjugate (Verdeyen, 1995).

The only term remaining is the amplitude term which could be represented in terms of position vector (r) and phase parameter $(\omega_{(z)})$, which is called the spot size, therefore it reduces to:

$$\delta_{01} = 1 - \frac{\int_0^{2\pi}\int_0^{\rho} r^2 e^{\frac{-2r^2}{\omega^2(z)}} dr d\phi}{\int_0^{2\pi}\int_0^{\infty} r^2 e^{\frac{-2r^2}{\omega^2(z)}} dr d\phi}\;..(9)$$

Solving for the numerator and denominator of equation (9), using the integration by part, one gets:

$$\delta_{01} = \left(1 + \frac{2\rho^2}{\omega^2_{(z)}}\right) e^{\frac{-2\rho^2}{\omega^2(z)}} \,..(10)$$

Here, one uses the following integrations values

$\int_0^{2\pi} d\phi = 2\pi,$

$\int_0^{\rho} r^2 e^{\frac{-2r^2}{\omega^2(z)}} dr = \frac{\omega^4(z)}{8r}\left(1 - \frac{2\rho^2}{\omega^2(z)}\right) e^{\frac{-2\rho^2}{\omega^2(z)}},$

$$\int_0^{\infty} r^2 e^{\frac{-2r^2}{\omega^2(z)}} dr = \frac{\omega^4(z)}{8r}$$

Substituting for $\left(\omega^2_{(z)} = \frac{\lambda L}{\pi} \frac{1}{\sqrt{1-g^2}}\right)$ and $\left(N = \frac{\rho^2}{\lambda L}\right)$ (Barriga et al., 2007) into equation (10), the diffraction loss becomes;

$$\delta_{01} = \left(1 + 2\pi N\sqrt{1-g^2}\right) e^{-2\pi N\sqrt{1-g^2}} \,..(11)$$

where N is Fresnel number and g – parameter is $\left[g = 1 - \frac{L}{R}\right]$, L is resonator length.

Equation (11) represents the diffraction loss in a symmetric resonator at transverse TEM_{01} mode.

It is clear from equation (11) that the quantity in the small parenthesis is deduced from fundamental definition of δ_{01} and the exponential term in the bracket is that of δ_{00} mode.

3. Results and Discussion

Equation (11) has been used to calculate diffraction loss in a symmetric resonator at transverse TEM_{01} mode for different g-parameters and Fresnel number values through fixed ranges allowable in gas lasers designed by T. Li (Li, 1965) and also for moderate values of Fresnel number taking (λ=632.8 nm), the resonator length and the aperture diameter are arbitrary constants of the resonator.

Figure (**3**) represents the values of diffraction loss for different values of g-parameter as [0.50, 0.80, 0.90, 0.95, and 0.99] respectively. The calculation and graphs were achieved from Tingye Li (Li, 1965) and using MATLAB programmers. Also remembering that all parameters ρ, $\omega_{(z)}$, L, R, and ω_0 are for a spherical symmetric resonator. But they are not used numerically because of canceled them. As diffraction loss and Fresnel number equations. Also, from figure (**3**), it is clear that the best agreement and coincidence of both results are observed from g-parameters of (0.80).

This closure or parallax of the experimental and theoretical values for the (δ_{01}) comes from that the direct dependence of (δ_{01}) on the (g) value charges from a resonator to another one, but it severely changes after they (g) value higher than 0.9 and reaching maximum at g = 0.99.

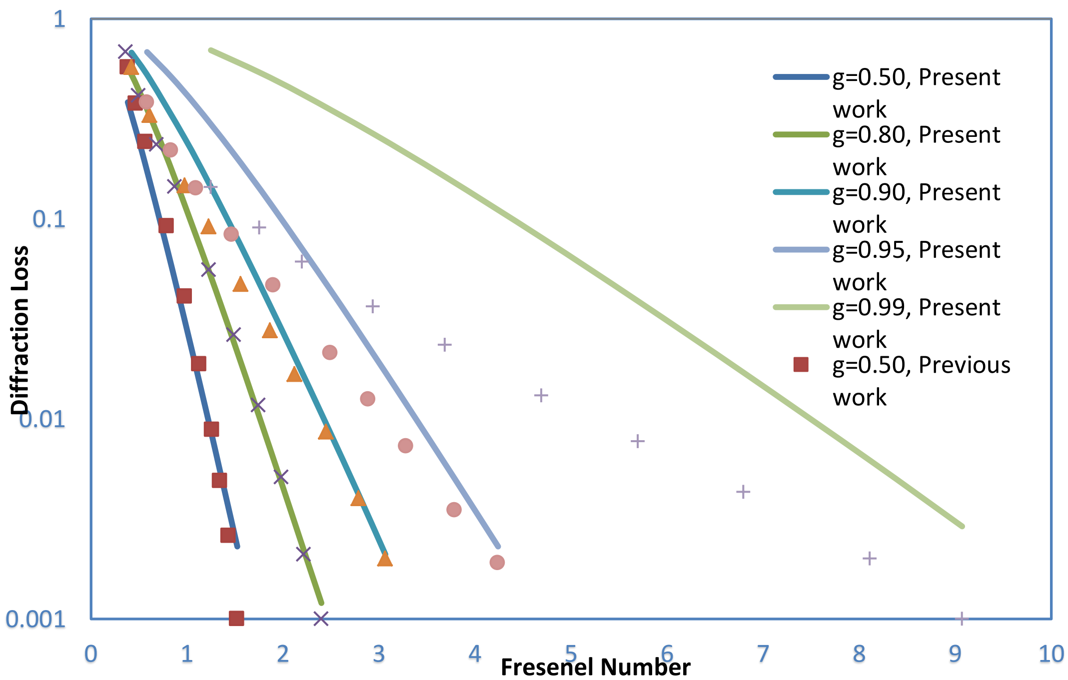

Figure (3): Diffraction loss versus Fresnel number for the TEM_{01} mode, for g=0.50, g=0.80, g=0.90, g=0.95 and g=0.99 for both works.

Figure (**4**) shows diffraction loss against different values of N – numbers. The values of both N and g are obtained as extension of the work done in Li maser design (Li, 1965). This is because of the high stability of this g-value.

Finally, it is observed that the optimum value of (g) as (0.80) is applicable in both lower-power laser and maser designs and also in different mode distribution especially in (TEM_{00}) and (TEM_{01}) modes and specifically when Fresnel number is calculated using Li data, whereas at this value the slope is given as (0.9556) which is excellent for mirrors reflectivity (R_2).

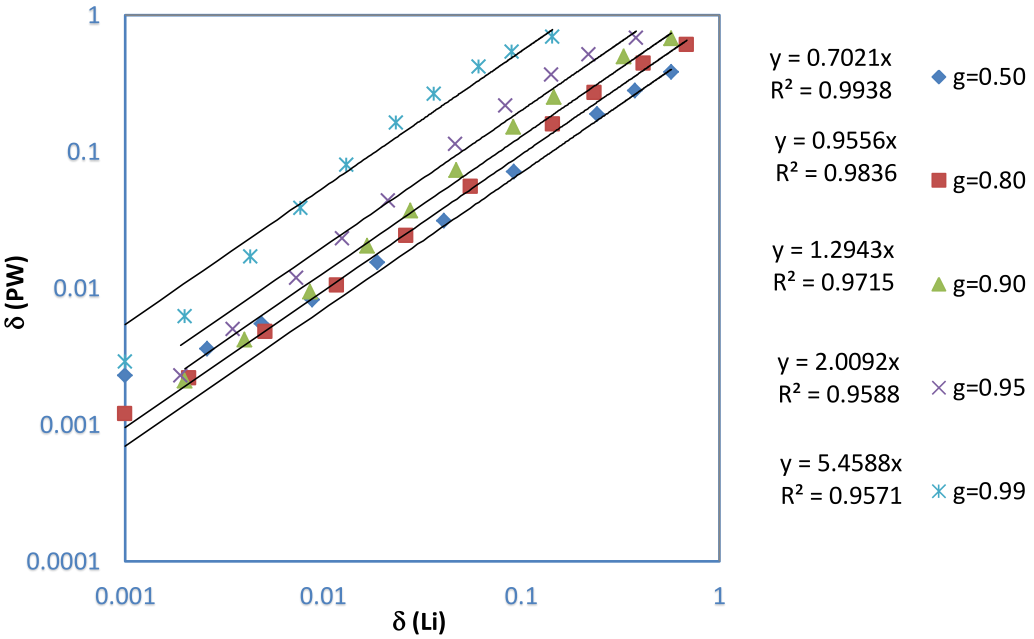

Figure (4): Variation of the diffraction loss for the present work with that of Li, for g=0.50, g=0.80, g=0.90, g=0.95 and g=0.99.

4. Conclusion

The diffraction losses observed in this work has a direct relationship with that of (TEM_{00}) modes, since beam characteristics in the resonator is stuck to the same parameters defined in Gaussian beam distribution. Thus, the parameter (N) has a direct effect on the losses on the surface of (M_2). The geometric shape of the cavity defines N and g parameters.

In this case, it is quite important to remember that the best fitting value of (g) is (0.8) in both theory and experiment tendencies throughout all values of (N). This is a fortunate result for (TEM_{01}) modes. Since, at higher values of (g) the losses take different category and it will be more complicated.

Also, it is clear that the extra-factor predicated in the theory for the losses in (TEM_{01}) mode was to be the addition of [$2\pi N\sqrt{1-g^2}$] to the result of (δ_{01}) mode with support Hermite polynomials as an approximation technique.

This is keyword of the solution for higher order modes at future also, since the arising factors are more complicated, but all of them depend on the well-known parameters (N), (g) which are in turn functions of λ, ρ, R, and L.

Finally, it is observed that the optimum value of (g) is (0.80) which is applicable in both lower-power laser and maser designs and also in different mode distribution especially in (TEM_{00}) and (TEM_{01}) modes and specifically when Fresnel number is calculated using Li data, where at this value the slope given as (0.9556) which is excellent for mirrors reflectivity (R_2).

5. References

AKSENOV, V. P., KOLOSOV, V. V., POGUTSA, C. E. & FILIMONOV, G. A. Dependence of the density of the probability of turbulent fluctuations of the orbital angular moment of a laser beam on the size of the receiver aperture. 26th International Symposium on Atmospheric and Ocean Optics, Atmospheric Physics, 2020. International Society for Optics and Photonics, 1156029.

AST, S., DI PACE, S., MILLO, J., PICHOT, M., TURCONI, M., CHRISTENSEN, N. & CHAIBI, W. 2021. Higher-order Hermite-Gauss modes for gravitational waves detection. *Physical Review D,* 103**,** 042008.

BARRIGA, P., BHAWAL, B., JU, L. & BLAIR, D. G. 2007. Numerical calculations of diffraction losses in advanced interferometric gravitational wave detectors. *JOSA A,* 24**,** 1731-1741.

BHAWAL, B. 2005. Diffraction losses of various modes in advanced LIGO arm cavity. *LIGO Tech. Note, LIGOT050234-00-E (LIGO, 2005)*.

BISTI, F., ALEXEEV, I., LANZETTA, M. & SCHMIDT, M. 2021. A simple laser beam characterization apparatus based on imaging. *Journal of applied research and technology,* 19**,** 98-116.

BORNEIS, S., LAŠTOVIČKA, T., SOKOL, M., JEONG, T.-M., CONDAMINE, F., RENNER, O., TIKHONCHUK, V., BOHLIN, H., FAJSTAVR, A. & HERNANDEZ, J.-C. 2021. Design, installation and commissioning of the ELI-Beamlines high-power, high-repetition rate HAPLS laser beam transport system to P3. *High Power Laser Science and Engineering,* 9.

COLDREN, L. A. 2021. Review of key vertical-cavity laser and modulator advances enabled by advanced MBE technology. *Journal of Vacuum Science & Technology A: Vacuum, Surfaces, and Films,* 39**,** 010801.

FALCÓN, R. E.-C., ESCUELA, A. M., SENDRA, J. S. & SANTANA, D. F. 2019. Subsampling techniques for gaussian laser beam radius estimation using a configurable quadrant photodiode (CQD). *Optics & Laser Technology,* 109**,** 412-417.

GEORGE, A., WEBER, H. J. & HARRIS, F. 2013. Mathematical Methods for Physicists. A Comprehensive Guide. Academic Press, Elsevier.

HASHEMIAN, S. & AKOU, H. 2020. Hollow high-quality electron beam generation by cyclotron autoresonance in laser-induced acceleration. *Optik,* 202**,** 163708.

HU, X.-Q., LEI, X.-C., SALSAC, A.-V. & BARTHÈS-BIESEL, D. 2020. Minuet motion of a pair of capsules interacting in simple shear flow. *Journal of Fluid Mechanics,* 892.

KARBSTEIN, F. 2018. Vacuum birefringence in the head-on collision of x-ray free-electron laser and optical high-intensity laser pulses. *Physical Review D,* 98**,** 056010.

KOGELNIK, H. & LI, T. 1966. Laser beams and resonators. *Applied optics,* 5**,** 1550-1567.

LAKHOTIA, H., KIM, H., ZHAN, M., HU, S., MENG, S. & GOULIELMAKIS, E. 2020. Laser picoscopy of valence electrons in solids. *Nature,* 583**,** 55-59.

LI, T. 1965. Diffraction loss and selection of modes in maser resonators with circular mirrors. *The Bell System Technical Journal,* 44**,** 917-932.

SASNETT, M. 2020. Propagation of multimode laser beams–the M2 factor. *The physics and technology of laser resonators.* CRC Press.

SIEGMAN, A. E. 1986. Lasers. Mill Valley, CA: Univ. *Science***,** 298-301.

SOHN, I.-B., CHOI, H.-K., NOH, Y.-C., KIM, J. & AHSAN, M. S. 2019. Laser assisted fabrication of micro-lens array and characterization of their beam shaping property. *Applied Surface Science,* 479**,** 375-385.

SVELTO, O. & HANNA, D. C. 1998. *Principles of lasers,* Springer.

URUNKAR, T., PATIL, S., VALKUNDE, A., VHANMORE, B., GAVADE, K. & TAKALE, M. 2018. Effect of Critical Beam Radius on Self-focusing of cosh-Gaussian Laser Beams in Collisionless Magnetized Plasma. *Communications in Theoretical Physics,* 70**,** 220.

VERDEYEN, J. T. 1995. Laser Electronics, 1995. Prentice Hall, Inc.

WANG, M., MA, Y., SHENG, Q., HE, X., LIU, J., SHI, W., YAO, J. & OMATSU, T. 2021. Laguerre-Gaussian beam generation via enhanced intracavity spherical aberration. *Optics Express,* 29**,** 27783-27790.

ZHOU, G., JI, Z. & RU, G. 2018. Complete Analytical Expression of Lorentz-Hermite-Gauss Laser Beams. *Lasers in Engineering (Old City Publishing),* 40.

ZHU, M., XU, N., ROSHANZADEH, B., BOYD, S., RUDOLPH, W., CHAI, Y. & SHAO, J. 2020. Nanolaminate-based design for UV laser mirror coatings. *Light: Science & Applications,* 9**,** 1-6.

ZHU, X., WU, D., WANG, Y., YU, F., LI, Q., QI, Y., KNIGHT, J., CHEN, S. & HU, L. 2021. Delivery of CW laser power up to 300 watts at 1080 nm by an uncooled low-loss anti-resonant hollow-core fiber. *Optics Express,* 29**,** 1492-1501.

Conflict of Interest

The authors confirm that there is no conflict of interest.