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Modeling Non-Homogenous Poisson Process and Estimating the Intensity Function for Earthquake Occurrences in Iraq using Simulation for data from January 1st 2018 to April 30th 2023

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ABSTRACT

The Non-homogeneous Poisson Process, with time-dependent intensity functions, is commonly used to model the scenario of counting the number of events that appear to occur in a given time interval. The identification of the process relies on the functional form of the intensity function, which can be difficult to determine. In this paper, a Non-Homogenous Poisson Process model is proposed to predict the intensity function for the number of earthquake occurrences in Iraq; the constructed model allows anticipating the number of earthquakes that occur at any time interval with a specific time length. Then, to estimate the model parameters, the data obtained from the annual reports of the Iraqi Meteorological Organization and Seismology (IMOS) from January 1^{st} 2018 to April 30^{th} 2023 are used. Moreover, a simulation study is conducted and a new algorithm is introduced to show the performance and applicability of the proposed model.

1.Introduction

Poisson process models have been widely used to describe the scenario of counting the number of events that occur at a certain rate in which the intensity function is formulated as a time-dependent variable. The Model with a constant rate of occurrence, which is known as Homogenous Poisson Process (HPP), does not allow for any changes in the intensity function of occurrence over a time interval. While a Non-Homogeneous Poisson Process (NHPP) is a more complex model; it is used in cases when events occur differently within time and the intensity function is not constant anymore. In addition, for the initial state, the probability of no events occurring is one and the probability of no events occurring is zero.

Destructive Earthquake Occurrences (EOs) are one of the biggest natural disasters that occur in various countries in the universe, apart from their devastating impact on people's lives, earthquakes can have a serious effect on economic collapse, and the cultural heritage of urban regions. Thus, modeling the EO is always a major research object in seismic probabilistic prediction and various modeling approaches have been developed from different viewpoints, for example: Stochastic Time-Predictable Model for EO (Anagnos and Kiremidjian, 1984); a Non-parametric hazard model to characterize the spatio-temporal occurrence of large earthquakes (Faenza, et. al, 2003); A statistical analysis and comparison of earthquakes and tsunami in Japan and Indonesia (Parwanto and Oyama, 2014). furthermore, to model the casualty rate of earthquakes and related covariates, linear regression, beta regression, and semi-parametric additive regression are used (Turkan and Ozel, 2014); testing probabilistic earthquake forecasting and seismic hazard models (Marzocchi and Jordan, 2018; Masoud, et. al, 2022); Generalized Linear Model for estimating the probability of EO (Noora, 2019) and many others. In addition, many stochastic approaches have been used as powerful tools for modelling EO, e.g., statistical models for earthquake occurrence and residual analysis for point processes (Ogata, 1988) ; A new approach, which is obtained by mixing Poisson distribution with quasi-xgamma distribution, to model the counts of earthquakes (Altun, at el., 2021). Despite that, scientists have never predicted a major earthquake but can only determine the probability that a significant earthquake will occur in a specific area within a certain time interval. Therefore, to estimate the number of EOs that may occur in the future to predict the consequences of such destructive events, proposing statistical models to describe the counts of Eos is very essential; it helps to take measures and actions based on the results obtained from implementing the proposed models.

In Iraq, despite the recognition of medium hazard levels and the potential for significant destruction caused by earthquakes, there remains a shortage of data and research studies dedicated to assessing future earthquake occurrences in the region. Several factors may contribute to this lack of data and researches. However, Many authors investigated the seismicity, which is the occurrence or frequency of earthquakes in different regions of Iraq (e.g. Al-Abbasi and Fahmi, 1985; Alheety, 2016; Khagoory and Al-Rahim, 2023; Kagan, 2010; Francizek, 2018) and many others. In this regard, there is a gap in the existing literature regarding the specific focus on modeling earthquake occurrences (EO) framework for informing decision-making processes related to earthquake preparedness and emergency response planning in the affected areas. Therefore, the main objective of this work is to develop a modeling approach for earthquake occurrences (EO) in Iraq and border regions using a Non-Homogeneous Poisson Process (NHPP) and estimating parameters of the intensity function. The focus is primarily on constructing a statistical framework for modeling earthquake occurrences over time, without delving into the geophysical causes underlying seismic activity. This paper is organized as follows: after introduction, in section1, the main definitions and theorems concerning the HPP and NHPP are introduced. In section 2, Statistical Analysis for the Data Source under Study is provided and graphical representations for the data are displayed. Then, the new proposed mode is introduced and the main parameters are estimated. To assess the model fitting, a simulation study is conducted and introduced in section 4. Finally, in section 5, the main conclusion is summarized.

Before introducing this model, the basic concepts and the main properties of Poisson Process and NHPP are described in addition to the main assumptions (Ross, 2007).

1.1 Poisson Process

In order to introduce a stochastic process known as Poisson Process and then to derive the model formula, the following are introduced:

1.1.1 Counting Process

A stochastic process $\{N(t), t \geq 0\}$, $N(t)$ indicates the total number of occurrences that take place by time t, is a counting process if the following are satisfied:

1. $N(t) \geq 0$;
2. $N(t)$ is integer valued;
3. If $s < t, then\ N(s) \leq N(t), s, t > 0$
4. For $s < t$, $N(t) - N(s)$ represents the total number of occurrences that take place in $(s, t]$.

The Poisson process is considered as the most important type of counting process, but the weakness of this process is its assumption that events are just as likely to occur in all intervals of equal size. A NHPP is a generalization, which reduces this assumption and represents a non-stationary process for the number of events that occur at time t with intensity function denoted by $\lambda(t)$. This model allows predicting the number of event occurrences at any time interval with a specific time length. These processes are defined below:

Definition 1

A counting process $\{N(t), t \geq 0\}$, is called a Poisson Process with rate of occurrence $\lambda > 0$ if the following are satisfied:

1. $N(0)$= 0; no occurrence for the initial state;
2. $\{N(t), t \geq 0\}$ has independent increments, or the number of events that appear to occur in the disjoint time intervals are independent;
3. $P\{[N(t+h) - N(t)] = 1\} = \lambda h + o(h)$; means that probability that one event will occur in a small interval depends on the length of the time interval and does not depend on the number of events that will occur outside the time interval (on the history of the process).
4. $P\{[N(t+h) - N(t)] \geq 2\} = o(h)$; this indicates that Probabilities that more than one event that occurs in a small-time interval can be ignored.

From the above assumptions, the following theorem is derived:

Theorem 1:

If $\{N(t), t \geq 0\}$ is a Poisson process with rate of occurrence λ>0, then for all $s, t > 0$, $[N(s + t) - N(s)]$ is a Poisson random variable with mean λt. That is, the number of events in any interval of length t is a Poisson random variable with mean λt (Ross, 2007).

1.1.2 Arrival and Interarrival Times:

For a Poisson process $N(t)$ with the rate of occurrence λ, if T_1 is the time of the first event that appear to happen and T_n is the time between the (n − 1) and the nth event, then $\{T_n, n = 1,2, \dots\}$ is called the sequence of interarrival times and $T_i, i = 1,2, \dots$; these are independent and identically distributed exponential random variables having mean 1/λ.

1.2 Non-Homogenous Poisson Process:

In this section, the NHPP, which is a generalization of the Poisson Process obtained by allowing the occurrence rate or intensity function at time t to be a function of t, is introduced: it is defined as follows:

Definition 2

The counting process $\{N(t), t \geq 0\}$ is defined as NHPP having intensity function $\lambda(t)$, $t \geq 0$ if the following are satisfied:

1. $N(0)$= 0;
2. $\{N(t), t \geq 0\}$ has independent increments;
3. $P\{[N(t+h) - N(t)] = 1\} = \lambda(t).h + o(h)$;
4. $P\{[N(t+h) - N(t)] \geq 2\} = o(h)$.

In addition, the expected value for the number of occurrences through time t, which is known as the mean cumulative function denoted by $m(t)$, is defined as $m(t) = \int_0^t \lambda(u)du$. It is obtained from the following theorem:

Theorem 2:

If a Poisson Process is non-homogenous with $\lambda(t)$, $t \geq 0$, then $[N(s + t) - N(s)]$ is a Poisson random variable with mean $(m(t+s)-m(s)) = \int_{s}^{t+s}\lambda(u)du$.

Thus, $m(t)=E[N(t)]$. As $N(t)$ is a non-decreasing step function, then the $m(t)$ must be non-decreasing function . When $m(t)$ is differentiable, $\frac{d}{dt}(m(t))$ can be expressed as the rate of change in the expected number of occurrences (Ross, 2007).

1.2.1 Intensity function

Let $N(t)$ denote the number of events occur in the interval $[0,t]$, and that $P_n(t)=P$, (n event occur in an interval $(0,t]$). The intensity function, which is a deterministic function for the number of occurrences that occur during an interval of length t and denoted by $\lambda(t)$, is the probability of occurrences in a small interval divided by the length of the interval; it is defined as:

$$\lambda(t) = \lim_{s\to 0}\frac{P[\{N(t+s)-N(t)\}\geq 1]}{s} \quad (1)$$

Thus, $\lambda(t)$ can be interpreted as the rate of change in the expected number of occurrences; there will be many events occur over intervals on which $\lambda(t)$ is large, and fewer occurrences over intervals on which $\lambda(t)$ is small.

On the basis of the above assumptions and definitions, for a NHPP, the probability that n events occur through the time interval $(0,t]$ is given by the following:

$$\Pr\{N(t)=n\} = \frac{(m(t))^n}{n!}\,e^{-m(t)}, \quad for\ t\geq 0, \quad (2)$$

The probabilistic behavior of the NHPP is completely defined by the mean-value function or the cumulative intensity function, which is interpreted as the expected number of events at time t, is defined as:

$$m(t)=E(N(t) = \int_0^t \lambda(u)du\ t\geq 0 \quad (3)$$

The variance $Var(t) = \int_0^t \lambda(u)du \quad t\geq 0$

(4)

Hence, the probability of exactly n events occurring in an interval $(t, t+s]$ is given by the following (Cinlar, 1975):

$$\Pr\{((N(t+s)-N(t)) = n\} = \frac{[\int_t^{t+s}\lambda(u)du]^n\, e^{-\int_t^{t+s}\lambda(u)du}}{n!} \quad \text{for } n=0,1,2,\ldots$$

(5)

Thus, $(N(t+s)-N(t)))$ is distributed as Poisson with an average value $(m(t+s)-m(t))$ with:

the probability of no event in the initial state or

$$\Pr\{N(t)=0\} = e^{-[m(t+s)-m(t)]} \quad (6)$$

When the time interval increases from $t\ to\ t+s$, then the average mean is defined as

$$m(t,s) = E((N(t+s)-N(t)) = \int_t^{t+s}\lambda(u)du,\quad t\geq 0 \quad (7)$$

$$Var(t,s) = \int_t^{t+s}\lambda(u)du \quad t\geq 0 \quad (8)$$

Therefore, based on the above assumptions, a NHPP model can be defined such that it provides concepts that allow to build a probabilistic model concerning incidents or event occurrences over time. When the NHPP model is constructed, it is essential to estimate both the unknown parameters and the functional form of the intensity function; this function is usually determined based on the initial description of the data under investigation.

In this paper, for modeling the data under study, a NHPP, which is often suggested as an appropriate model when a system whose rate varies over time, is proposed. This model is characterized by an intensity function to describe the process changes over time. To estimate this function, the data under study is used: the data represents the number of earthquake occurrences happened in Iraq and its boarders from January 1st 2018 to April 30th 2023.

2. Statistical Analysis for the Data Source under Study

The following data, which is shown in Table 1, provides information relevant to the number of earthquake occurrences happening in Iraq and its border from January 1st 2018 to May 1st 2023; the data is reported by the Iraqi Meteorological Organization and Seismology (IMOS). For analyzing data under study, the Statistical Package for Social Sciences (SPSS) software has been used and the results are shown by the following table:

Table 1: The number of earthquake occurrence in Iraq and its border from January 1st 2018 to April 30th 2023.

Year	Jan .	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2018	25	41	27	30	18	11	4	23	23	18	36	24	280
2019	55	4	3	61	11	11	9	16	11	6	5	2	194
2020	19	8	9	7	2	13	8	12	6	1	1	7	93
2021	31	29	19	49	19	37	1	25	20	32	22	13	297
2022	19	29	28	24	39	37	27	27	28	19	17	20	314
2023	18	18	24	19									79

Table 1 shows the number of occurrences for the years 2018 to April, 2023; it is clear that Earthquakes in Iraq, which is categorized as medium hazards, tend to happen occasionally or at irregular intervals due to the country's geographical positioning. Only the eastern and northeast regions directly experience seismic activity from the Tauros-Zagros structural zone. Consequently, the occurrence of earthquakes is restricted in number, resulting in a lower perceived risk level. Additionally, the data indicates that the total occurrences in 2022 exceed those of other periods, while the lowest number of occurrences was recorded in 2020.

The following figure, Figure 1, illustrates the frequency distribution for the data set under study.

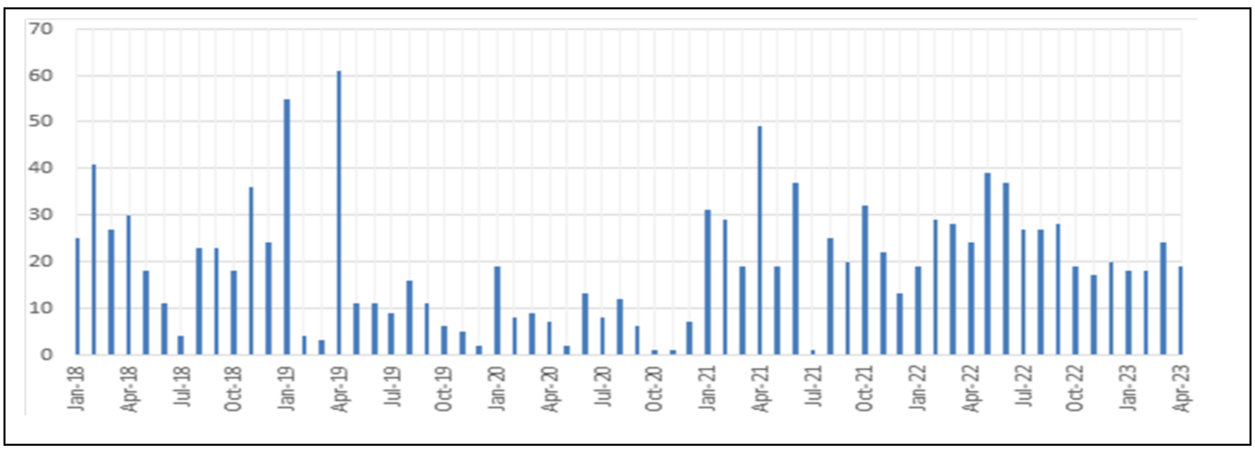

Figure 1: The frequency distribution for the number of earthquakes occurred in Iraq.

Moreover, based on the intensity function, the data set is explained by the following figure, Figure 2.

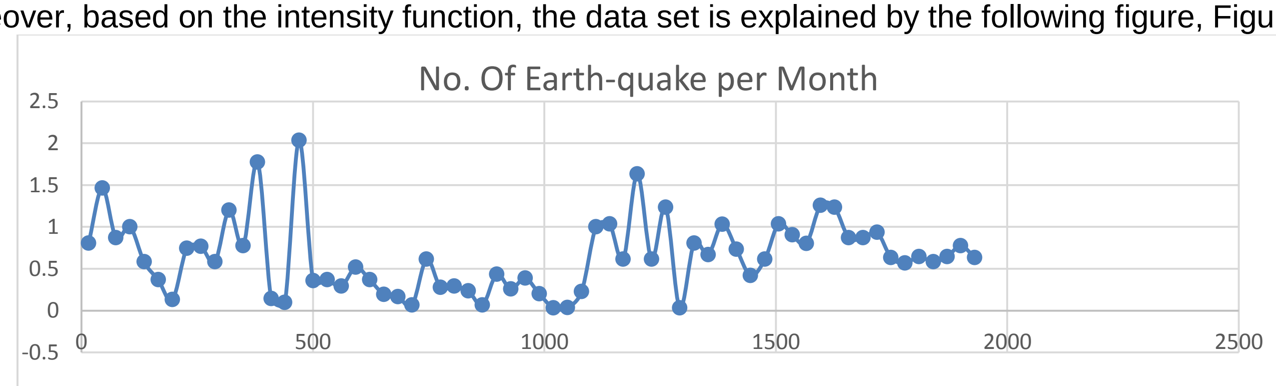

Figure 2: The number of earthquakes with the intensity function.

Therefore, considering the context of the described issue, this study utilized a NHPP (Non-Homogeneous Poisson Process) to quantify the occurrences of earthquakes in Iraq and subsequently predict their likelihood.

3. Model Description:

The objective is to present a model with the fewest variables to sufficiently describe the dataset. This paper introduces a Non-Homogeneous Poisson Process (NHPP) as a proposed method for modeling the dataset under examination. The NHPP is detailed below:

3.1 The Proposed NHPP Model

In this section, to study the number of earthquakes that occur in Iraq and its boarders over a time interval, say (0,T], a NHPP is proposed. As the intensity function for this process plays a crucial role in characterizing the probabilistic behavior of the NHPP, the main concern is the identification of the functional form of the function, which may take various forms, and then the estimation of its parameters; the parameters are usually estimated based on the initial description of the data under investigation.

If it is assumed that a counting process $\{N(t), t \geq 0\}$, which counts the total number of events that have occurred up to time T, takes values on {0,1,2,…}, each value represents the number of earthquake occurrences in a time interval (0,T]. Due to the nature of these occurrences, pre-assumption that it is a NHPP with some parameter, $\lambda(t) > 0$ seems to be justified.

Therefore, a NHPP is proposed to model the number of earthquakes that occur over a time intervals in Iraq and its boarders; it is characterized by a deterministic intensity function $\lambda(t)$ describing how the occurrence rate changes up to time T , where the time is measured on a daily scale.

Then, $N(t)$ is defined as a NHPP if $\Pr\{N(t) = n\} = \frac{(m(t))^n}{n!}\, e^{-m(t)}$, with the expected number of events of NHPP over time interval [0, T], $m(t)$. i.e:

$E\big(N(t)\big) = m(t)$ where $m(t) = \int_0^t \lambda(u)du$

Hence, $E((N(t+s) - N(t))$, which defines the expected value of the increment for the process, can be determined from equation (7). These rules can be implemented empirically if the intensity function $\lambda(t) > 0$ is known. To define this function, the data from the annual reports of the IMOS from January 1st 2018 to May 1st 2023 were used. The statistical analysis of the data, which is described by Table 1, shows that the intensity function $\lambda(t)$ can be approximated by the simplest deterministic mathematical function known as the simple linear regression defined as:

$Y = \alpha + \beta X$

(9)

Hence, the constructed model allows predicting the number of event occurrences at any time interval with a time length t when the parameter values are estimated. Numerous estimation methods have been used to predict parameters; the least squares method is a standard approach, which is used in this study, which is described below.

3.2 Parameter Estimation for the Model:

Consider a NHPP with intensity function $\lambda(t)$, which is assumed to be nonnegative, defined by equation (9) over a fixed interval $(0, T]$, $0 < t_1 < t_2, \cdots, < t_{n-1} < t_n \leq T$. The objective is to estimate parameters for the function λ,(t) using the least squares. The concept of least squares, which is a method for estimating the parameters of a statistical model, is fitted to sample data by minimizing an appropriate sum of squared estimation errors. The empirical intensity function is approximated by the Simple Linear Regression Model described by equation (9) that satisfies the following:

$S(\alpha, \beta) = \sum_1^n (y_i - (\alpha + \beta x_i))^2$

(10)

Then, the above equation is minimized to estimate each of α, β (Singh and Masuku, 2013). The least squares estimate of the intercept α and β of the true regression line is given by:

$\hat{\beta} = \frac{\sum_1^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_1^n (x_i - \bar{x})^2}$, $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$

(11)

Hence, the predicted intensity function is defined as $\hat{y} = \hat{\alpha} + \hat{\beta}x$ (12)

Then, by using the annual data reported by the IMOS in Iraq, which is explained by table 1, then by applying equation (11), the estimates for the two parameters are: $\hat{\alpha} = 0.59,\ and\ \hat{\beta} = 0.00006$

Thus, by using the data set given by Table 1 applied to equation (12), the following model is resulted:

$\hat{y} = 0.59 + 0.00006x$ (13)

The following figure, Figure 3, shows the fitted model and the data representation for the cumulative intensity function:

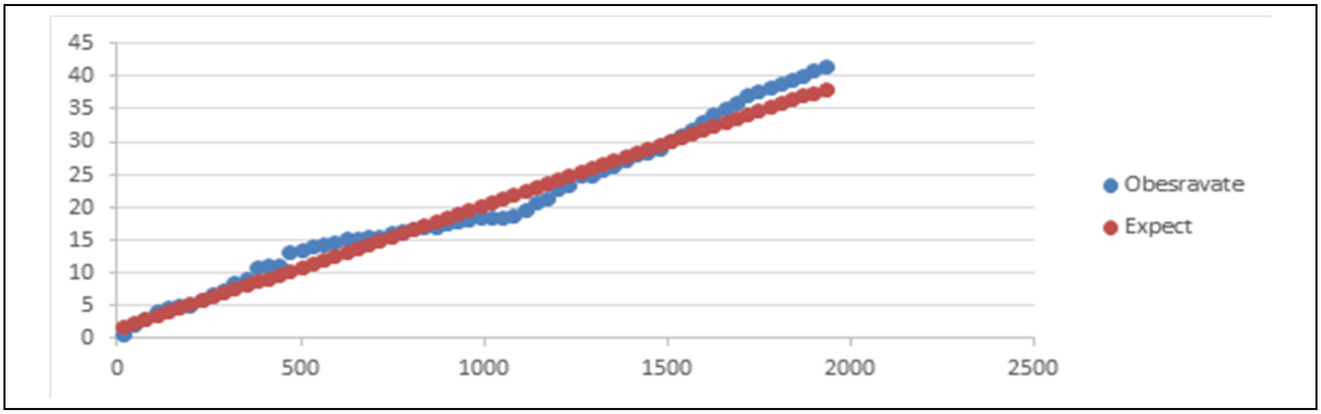

Figure 3: The observed and expected fit for the interval time and cumulative intensity.

4. Simulation

In this section, to assess the model fitting, a simulation study is conducted. The cumulative number of occurrences $N(t)$ for the NHPP is generated by considering equally spaced intervals over the time range $(0, T]$; the process of generating random variables is repeated for 30 times. Then, the same procedure is applied to different forms of intensity function $\lambda(t)$; each with different values of $\alpha \; and \; \beta$. Finally, based on the results obtained from implementing the simulation procedure each of $\alpha \; and \; \beta$ are estimated using equation (11), which is an estimated procedure for estimating the parameters for the regression model that describes the behavior of the intensity function. The proposed algorithm is different than the rest of the algorithms written for simulating a NHPP (Lewis and Shedler, 1979); equally space intervals are considered, so there is no need to simulate time intervals but it generates the number of occurrences that occur in each interval. For this algorithm, Matlab code is used and then to estimate $\lambda(t)$, SPSS (Statistical Package for Social Sciences) is implemented. The algorithm describing the main steps for simulating a NHPP is introduced below.

4.1 Generating the number of occurrences for the NHPP

Assume that N(t) is the cumulative number of occurrences of a NHPP with intensity function described by $\lambda(t) = \alpha + \beta t$ for $t < T,$ then to simulate N(t) the following algorithm is defined:

Algorithm

Step 1: let j=1

Step 2: Set initial value for $\lambda = 1$; Let $T = N; \; t = 0;, \; i = 0;$

Step 3: Generate random number $u \sim U(0,1);$

Step 4: Compute $t = t - \frac{1}{\lambda} ln(u)$, if $t > T$ then go to step 7;

Step 5: $Let \; i = i + 1; K(j, i) = t;$

Step 6: Set $\lambda = \alpha + \beta * t$: go to step 3;

Step 7: Let $j = j + 1$: $if \; j <= 30$ go to step 2;

Step 8: Set $N(t) =$ average for each row of event array K;

Step 9: end

4.2 Simulation results and discussion

The following figures, Figures 4, 5, 6, show the implementation of the algorithm for simulating the number of occurrences of a NHPP with intensity function $\lambda(t) = \alpha + \beta t$, for different values of T and α, β.

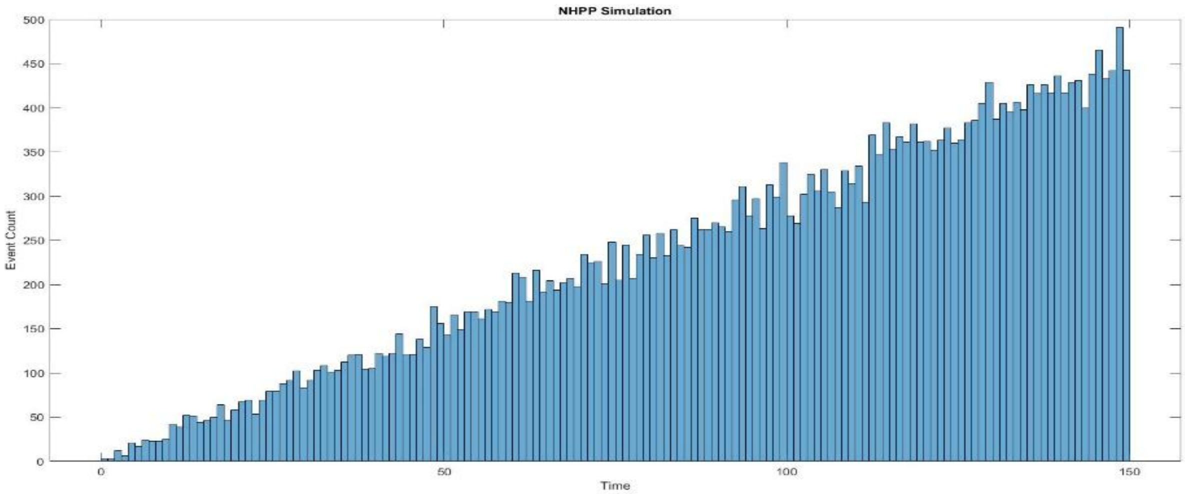

Figure 4. Number of occurrence *N(t)* for $\lambda(t) = 2 + 3t$ for $0 < t < 150$

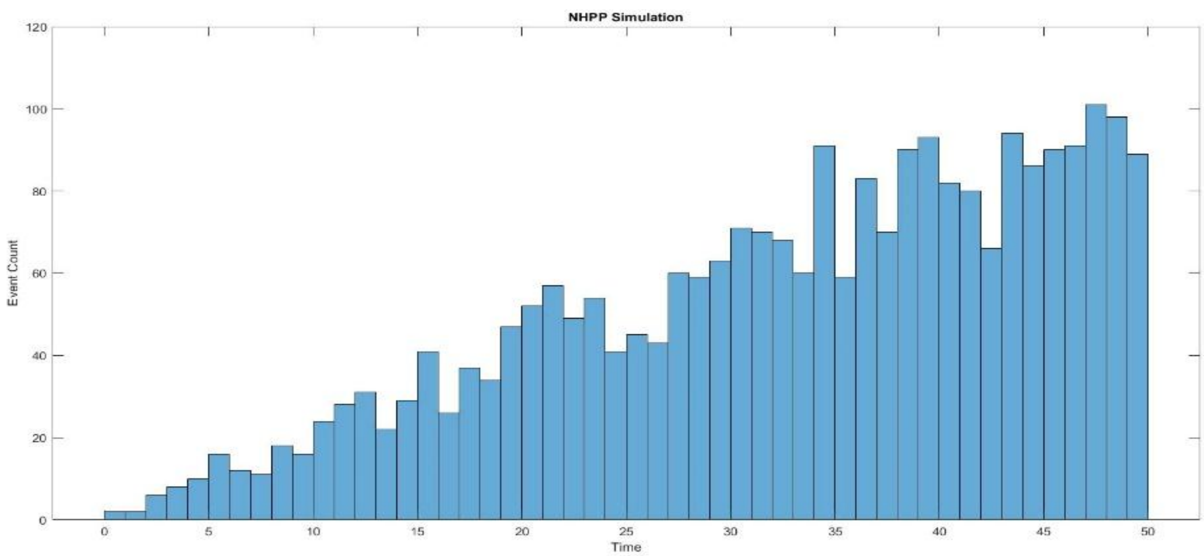

Figure 5. Number of occurrence *N(t)* for $\lambda(t) = 1 + 2t$ for $0 < t < 50$

Thus, figures above, Figure 4 and Figure 5, show that the estimated intensity functions $\lambda(t)$, which are resulted from simulation process for different values of α and β, can be approximated by a simple linear regression model.

The following table, Table 2, displays the results for generating $N(t)$ for the time interval: $0 < t < 150$, for different functional forms for $\lambda(t)$, (with different values for α and β); from the regression model, based on simulated data, each of α *and* β are estimated. Finally, the mean square error (MSE) for the observed and estimate model is obtained.

Table 2: Parameter estimation based on simulated data with Mean Square Error

α *and* β	$\hat{\alpha}$ *and* $\hat{\beta}$	MSE
$\alpha = 2$ *and* $\beta = 3$	$\hat{\alpha} = 0.675$ *and* $\hat{\beta} = 3.001$	7.359338
$\alpha = 3$ *and* $\beta = 2$	$\hat{\alpha} = 2.356$ *and* $\hat{\beta} = 1.994$	4.972138
$\alpha = 0$ *and* $\beta = 1.5$	$\hat{\beta} = 1.443$	3.925877
$\alpha = 0$ *and* $\beta = 4$	$\hat{\beta} = 3.843$	10.18737

4.3 Modeling the number of occurrences based on real data and predictions

Therefore, the proposed NHPP model is shown to be an appropriate model for predicting the number of earthquake occurrence and hence it will be possible to estimate the expected number of earthquakes that will occur in any time interval in which the cumulative number of EO in Iraq and its boarders is determined using equation (12) and then the linear intensity function for EO is obtained as:

$$\hat{\lambda}(t) = \hat{\alpha} + \hat{\beta}t = 0.59 + 0.00006t \quad (13)$$

When equation (3) is applied to the data set, the $E(N(t))$ by time t, which is defined as $m(t)$, is $m(t) = \int_0^t (\hat{\alpha} + \hat{\beta}u)du = \hat{\alpha}t + \hat{\beta}t^2$, with $\hat{\alpha} = 0.59$; $\hat{\beta} = 6 * 10^{-5}$

Hence, $m(t) = 0.59t + 0.00006t^2$ (14)

Therefore, modeling the number of earthquakes that occur in Iraq can be considered as a NHPP with parameters $m(t) = 0.59t + 0.00006t^2$ for $t > 0$;

$\Pr\{N(t) = n\} = \frac{(m(t))^n}{n!} e^{-m(t)}$ which implies that for the data under study this becomes:

$$\Pr\{N(t) = n\} = \frac{(0.59t + 0.00006t^2)^n}{n!} e^{-(0.59t + 0.00006t^2)}$$

Then, from equation (5), the number of earthquakes at a time interval with the length of the interval s, can be predicted as follows:

$$\Pr\{((N(t+s) - N(t)) = n\} = (m(t+s) - m(t))^n e^{-((m(t+s)-m(t))}/n!$$

For example, if it is required to predict the expected number of earthquakes that will occur from 1 to 7 November 2023, as in equation (7), can be anticipated as follows:

For the data set under study, the time interval $[1915,1945)$ was reported as the last interval for the data up to April 2023, then the time interval for 1 to 7 December 2023 will be $[2160, 2190)$. Then, the length of the interval is 6 at time $t = 2160$.

$$E((N(2166) - N(2160)) = \int_{2160}^{2166} 0.59 + 0.00006t\, dt = 0.59t + 0.00006t^2 \Big|_{2160}^{2166} = 4.3186$$

$$\sigma(t,s) = \int_t^{t+s} \lambda(u)du = \sqrt{E((N(2166) - N(2160))} = \sqrt{4.3186} = 2.07$$

Thus, by implementing the proposed model defined by a NHPP with $\lambda(t)$ to the data under study, the expected number of earthquakes that will occur in any time interval can be predicted. The above application is an example that in the first week of December 2023, it is expected that about 4 earthquakes will occur in Iraq and its borders.

5. Conclusion

The main objective of this work is to develop a modeling approach for earthquake occurrences (EO) in Iraq and border regions using a Non-Homogeneous Poisson Process (NHPP). The focus is primarily on constructing a statistical framework for modeling EO over time and to estimate the number of EOs that may occur in the future to predict the consequences of such destructive events. In this paper, a NHPP model has been used to model the number of earthquakes that occur in Iraq by defining the functional form of λ(t). Then, to estimate its parameters by using least square method, the data obtained from the annual reports of the Iraqi Meteorological Organization and Seismology (IMOS) from January 1st 2018 to April 30st 2023 are employed. Moreover, to assess the model fitting, a simulation study is conducted and the obtained results show the performance and applicability of the proposed model. Finally, as an illustration, theoretical findings are utilized to forecast the anticipated number of earthquakes that could occur within Iraq and its borders for a designated time interval.

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