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Solving Stochastic Transportation Programming Problems with Fuzzy Information on Probability Distribution Space Using a New Approach

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ABSTRACT

A study is conducted on solving stochastic transportation linear programming problems with fuzzy uncertainty information on probability distribution space (STLPPFI) model problems. A proposed method utilizes the concepts of alpha-cut technique with truth degrees technique on probability distribution, linear fuzzy membership function (LFMF), linear fuzzy ranking function (LFRF), trapezoidal fuzzy number (T_pFN), triangular fuzzy number (T_rFN) and expectation weighted summation (EWS) technique. Those are used to convert STLPPFI into its corresponding equivalent deterministic transportation linear programming problem (DTLPP) via defuzzifying fuzziness on probability distribution and derandomization randomness of problem formulation respectively. Although, matrix minima cost method (MMCM) with modify distribution method (MODI) respectively are used on obtained DTLPP to get optimal solution. In addition, decision maker (DM) manually decides which of resulting solution is a post optimal solution via choosing a solution that has suitable situation to DM. A proposed algorithm along with numerical example on electricity field illustrating the practicability of it. The obtained results with existing methods show the efficiency of strategy proposed solution method based on the analysis that from results performed.

1. Introduction

The DTLPPs are one of the most useful mathematical models, and used for predicting current real-life problems, where main goals of DTLPP are to minimize the total transport costs, increasing amount of the transporting goods/objects as possible, increasing the availability/production sources and decreasing the demand/requirement endpoints as possible by remove unnecessary points and reduce waste points. Then striking balance between availability/production sources and demand/requirement endpoints, since we could not solve DTLPP model problems without balance condition (Reeb, James Edmund;Leavengood, Scott A, 2002). The DTLPP has to recourse fuzziness and randomness to deals with most of real-life problems that motivate us to search for more efficient strategy solution for STLPPFI. Thereby, the concept of STLPPFI needs to solve the complicated problems in the transportation problems. The problem formulation has randomness in objective cost coefficients and fuzziness in linear inequalities polyhedral sets of information probability distribution space.

The linear programming problem (LPP) is an mathematical formulation programming problem which contain a singleton objective function that should be optimize (max/min) subject to a set of constraint equations/inequalities that should be satisfies, with non-negative unrestricted variable set, where the objective polynomial function with constraints equations/inequalities are linear and parameters of them are known with certainty known information on probability distribution space (Abdelaziz, F Ben;Masri, Hatem, 2005; Guo, Haiying;Wang, Xiaosheng;Zhou, Shaoling, 2015; Ameen, 2015). Depending on (Winston, Wayne L;Goldberg, Jeffrey B, 2004; Sharma, 1974; Sengamalaselvi, 2017) DTLPP is an LPPs with minimizing objective polynomial function or minimizing total transporting cost of shipping objects, subject to both availability and requirement constraints set where total availabilities satisfy total requirements, parameters set are non-negative, both objective function with constraints are linear and parameters of them are known with certainty known information on probability distribution space. An DTLPP is called a stochastic transportation linear programming problems (STLPP) when parameters in DTLPP are random and represented by uncertainty on probability distributions (Ameen, 2015; Hamadameen, Abdulqader Othman;Hassan, Nasruddin, 2018; Ben Abdelaziz, F;Masmoudi, Meryem, 2012). The probability distribution space $(\Omega, 2^{\Omega}, P)$ of an STLPP in many cases are unknown or undetermined/unspecified, since partial information on probability distribution are existed or any information on probability distribution does not have or information on probability distribution are fuzzy unknown or probability distribution are fuzzy distributed, then the probability distribution space should be specified/determined as first step in solution procedures. Moreover, STLPPFI is an STLPP's under fuzzy uncertainty unknown information on probability distribution and described by fuzzy linear inequalities polyhedral set (Ameen, 2015; Abdelaziz, Fouad Ben;Masri, Hatem, 2009; Hamadameen, Abdulqader Othman;Zainuddin, Zaitul Marlizawati, 2015).

This study focuses on the producing and distributing electricity power companies. The first main elementary problems created to a company are non-deterministic values of transporting cost coefficients via using fuel for producing electricity power, generator machines sizes, difference length in the distance between sources with endpoints, various kinds of amounts of using electricity in Kw and various kinds of users as home, medical area, industrial area, factory area, government offices. The second main elementary problems created to company are non-deterministic values of creation probability distribution parameters via various length of probability problem intervals as each problem interval on probability distribution space has weight rank in the break-down electricity grid system or has weight rank in importance elementary company planning: such as fuel, generator load-operating, distribution table-time, voltage transformer, distances, summer (winter) over-load problems, generator coolant system problems and so on. Furthermore, the company does not have previous information about sorting weighted ranks of probability distribution problems till getting experience in solving problems and/or how to work with problems and sort out problems via importance ranks of break-down system. The rest of this paper is structured as follows: in section two addresses the preliminaries of fuzzy concepts, polyhedral set types and literature review, followed by the mathematical formulation of STLPPFI and DTLPP in section three. Techniques for solving STLPPFI in section four, while a numerical example is solved in section five, post optimality and sensitivity analysis takes its place in section six, followed by explanation of the ranking function and its effectiveness in the uncertainty in section seven, and conclusions in section eight.

2. Preliminaries of Fuzzy Concepts, Polyhedral Set Types and Literature Review

This section shows some necessary definitions of elementary fuzzy concepts, then two kinds of polyhedral sets will be states that relate to certainty information probability distribution which are necessary to good understanding of fuzzy transformations, then reviews some related previous works to STLPPFI as follows:

2.1. Basic Definitions

In this section, some basic definitions which are necessity for the following sections are listed.

2.1.1. Definition: The Membership Function

Let X be a universal set, where fuzzy subset $\widetilde{A}$ of X as well as set H is a subset of fuzzy set's $\widetilde{A}$ (i.e., $H = [0,1] \subseteq \widetilde{A} \subseteq X$), so the membership function $\mu_{\widetilde{A}}$ of fuzzy set $\widetilde{A}$ defines the degrees of belongingness membership of elements $x \in \widetilde{A}$, which shows that degree of $x \in \widetilde{A}$, or it is a function that maps some set of real numbers to interval [0, 1]. Mathematically; $\mu_{\widetilde{A}}: X \to H, \forall x \in X$ or $\mu_{\widetilde{A}} \subseteq \widetilde{A} \times H$ (Sakawa, Fundamentals of fuzzy set theory, 1993; Mahdavi-Amiri;NezamNasseri;Seyed Hadi, 2007; Dharani, K;Selvi, D, 2018), as shown in Figure (2.1-1).

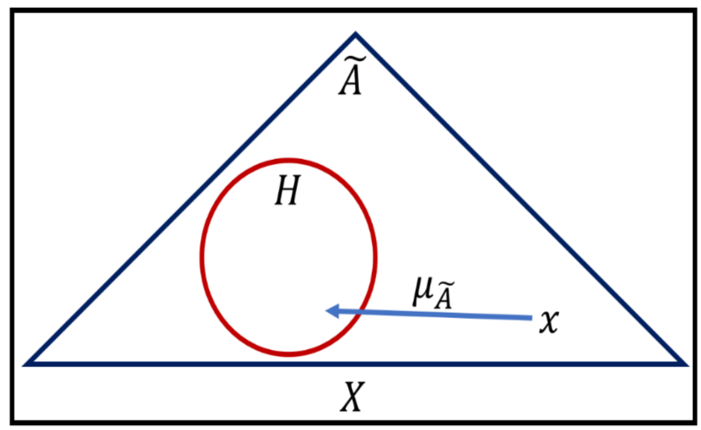

Figure 2.1-1. The Membership Function

2.1.2. Definition: The Fuzzy Set

Let X be a universal set, $\widetilde{A} \subseteq X$. $\widetilde{A}$ is called a fuzzy/non-exact set that contains ordered pairs, $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)), \forall x \in X\}$ where $\mu_{\widetilde{A}}(x)$ is membership function of $x \in \widetilde{A}$ (i.e., a characteristic/indicator function for $\widetilde{A}$ that shows to what degree $x \in \widetilde{A}$). If the height of fuzzy set is one, then fuzzy set is normal. Where the height of a fuzzy set is the largest membership value attained by any point in the set (Ameen, 2015; Sakawa, Interactive multiobjective linear programming with fuzzy parameters, 1993; Mahdavi-Amiri, N;Nasseri, SH, 2006). For example, formulation (4.2-3) is one of the fuzzy set kinds.

2.1.3. Definition: The Modal/Core and Support

The modal/core of a fuzzy set $\widetilde{A}$ of universal set X is the crisp subset of X i.e., $core(\widetilde{A}) \subseteq X$ that containing all elements have been membership grade of it is equal to one. Mathematically, $core(\widetilde{A}) = \{\forall x, \widetilde{A}(x) = 1 \wedge x \in X\}$. Where the support of a fuzzy set $\widetilde{A}$ of universal set X is a set of elements in X i.e., $supp(\widetilde{A}) \subseteq X$ that cause $\widetilde{A}(x)$ is positive. Mathematically, $supp(\widetilde{A}) = \{\forall x \in X; \mu_{\widetilde{A}}(x) > 0\}$ (Dharani, K;Selvi, D, 2018; Mahdavi-Amiri;NezamNasseri;Seyed Hadi, 2007; Sakawa, Fundamentals of fuzzy set theory, 1993).

2.1.4. Definition: Convexity of Fuzzy Number

Fuzzy number is a convex fuzzy set $\widetilde{A}$ on $\mathbb{R}$ if and only if the membership function of it is piecewise continuous, and there exist have three intervals $[a, b], [b, c]$ and $[c, d]$ such that $\widetilde{A}$ is increasing on $[a, b]$, equal to 1 on $[b, c]$, decreasing on $[c, d]$, and equal to 0 elsewhere, $\forall a, b, c, d \in \mathbb{R}$ (Ameen, 2015; Dharani, K;Selvi, D, 2018; Mahdavi-Amiri;NezamNasseri;Seyed Hadi, 2007; Sakawa, Fundamentals of fuzzy set theory, 1993). For example, in Figure (4.2-1) and formulation (4.2-4) shows seven different convex fuzzy numbers.

2.1.5. Definition: The Trapezoidal Fuzzy Number (T_pFN)

A T_pFN is $\widetilde{A} = (a^L, a^U, \alpha, \beta)$, where $[a^L, a^U]$ is the modal/core set of $\widetilde{A}$, and $[a^L - \alpha, a^U + \beta]$ is the support part set of $\widetilde{A}$ (Ameen, 2015; Dharani, K;Selvi, D, 2018; Mahdavi-Amiri;NezamNasseri;Seyed Hadi, 2007; Sakawa, Fundamentals of fuzzy set theory, 1993), as shown in Figure (2.1-2).

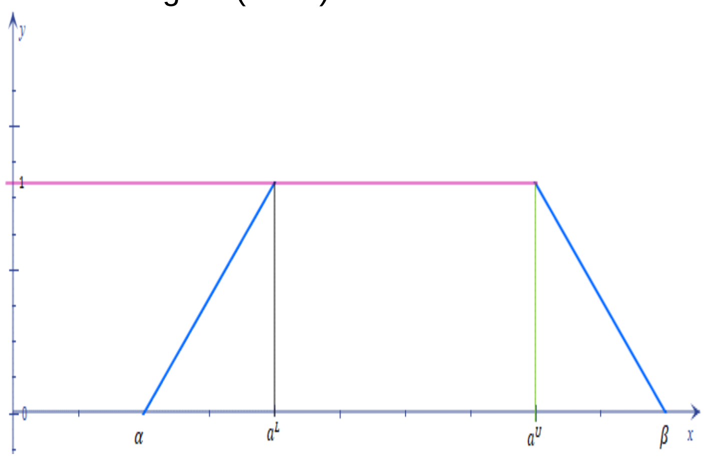

Figure 2.1-2. The T_pFN

where LFMF for T_pFN is as following:

$$\mu(x) = \begin{cases} \dfrac{x-\alpha}{a^L-\alpha} & \alpha \le x \le a^L \\ 1 & a^L \le x \le a^U \\ \dfrac{\beta - x}{\beta - a^U} & a^U \le x \le \beta \\ 0 & otherwise \end{cases}$$

Formulation 2.1-1. The LFMF of T_pFN

2.1.6. Definition: The Triangular Fuzzy Number (T_rFN)

The T_pFN is reduced to T_rFN and denoted by $\widetilde{A} = (a, \alpha, \beta)$, where $a = a^L = a^U \in \widetilde{A} \subseteq F(R)$ (Ameen, 2015; Dharani, K;Selvi, D, 2018), thus $\widetilde{A} = (a, \alpha, \beta) \subset (a^L, a^U, \alpha, \beta) \subseteq F(R)$, as shown in Figure (2.1-3).

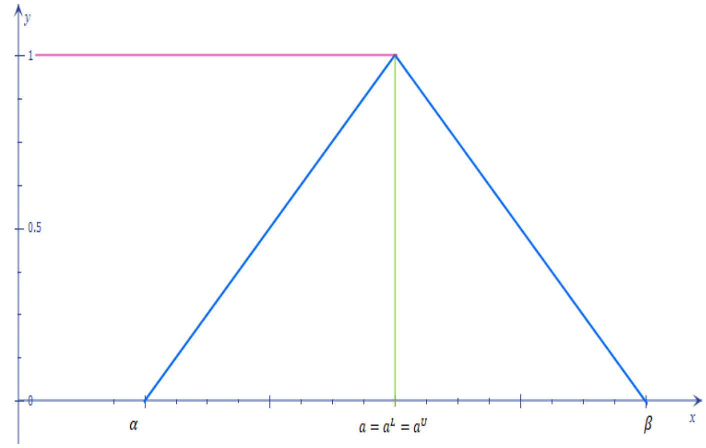

Figure 2.1-3. The T_rFN

where LFMF for T_rFN is as following:

$$\mu(x) = \begin{cases} \dfrac{x-\alpha}{a-\alpha} & \alpha \le x \le a \\ 1 & x = a \\ \dfrac{\beta - x}{\beta - a} & a \le x \le \beta \\ 0 & otherwise \end{cases}$$

Formulation 2.1-2. The LFMF of T_rFN

2.1.7. Definition: The Ranking Function $(R(F))$

A ranking function $R(F): F(R) \longrightarrow \mathbb{R}$ is a mapping that transforms each fuzzy number into its corresponding real value in real line, where a natural order exists (Ameen, 2015; Dharani, K;Selvi, D, 2018). For example, formulation (4.2-5) and formulation (4.2-6) are two kinds of ranking functions.

2.2. The Relationship Between Polyhedral Set Types with Uncertainty Unknown Information on Probability Distribution Space

This subsection is preferring two kinds of information on probability distribution which are fuzzy and stochastic polyhedral sets as follows:

2.2.1. Definition: The Fuzzy Polyhedral Set $\widetilde{\pi}$

Where the polyhedral set information of probability distribution space $(\Omega, 2^{\Omega}, P)$ was approximated on $\widetilde{\pi}$ and generated by fuzzy/inexact inequalities on π and non-crisp of P, which are for each probability p_i of a given event $\omega_i \in \widetilde{\pi}, i = 1,2, \dots, s$, or the uncertainty unknown information on probability distribution space $(\Omega, 2^{\Omega}, P)$ was fuzzy/non-exact on π, then it is called fuzzy polyhedral set and formed by:

$$\tilde{\pi} = \left\{ \begin{array}{c} p = (p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N; \\ Ap \precsim b; \sum_{i=1}^{N} p_i = 1; \\ \forall p_i \geq 0; i = 1, \dots, N \end{array} \right\}$$

Formulation 2.2-1. The Fuzzy Polyhedral Set $\tilde{\pi}$

where A and b are (s, N) and $(s, 1)$ dimensions fixed fuzzy random matrices respectively, and $\precsim$ was a fuzzy/inexact inequality and non-crisp of P which meant that Ap was almost equal or less than b (Abdelaziz, F Ben;Masri, Hatem, 2005; Abdelaziz, Fouad Ben;Masri, Hatem, 2010; Abdelaziz, 2012; Ameen, 2015).

2.2.2. Definition: The Stochastic Polyhedral Set π

Where the polyhedral set information on probability distribution space $(\Omega, 2^{\Omega}, P)$ was stochastic on π and generated by stochastic inequalities on π and crisp of P, which are for each probability p_i of a given events $\omega_i \in \pi, i = 1,2, \dots, s$, or the uncertainty unknown information on probability distribution space $(\Omega, 2^{\Omega}, P)$ was stochastic on π, or probability distribution space converted from fuzzy to stochastic, then it is called stochastic/default/known polyhedral set and formed by:

$$\pi = \left\{ \begin{array}{c} p = (p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N; \\ Ap \leq b; \sum_{i=1}^{N} p_i = 1; \\ \forall p_i \geq 0; i = 1, \dots, N \end{array} \right\}$$

Formulation 2.2-2. The Stochastic Polyhedral Set π

where A and b are (s, N) and $(s, 1)$ dimensions fixed random matrices respectively (Abdelaziz, F Ben;Masri, Hatem, 2005; Abdelaziz, Fouad Ben;Masri, Hatem, 2010; Ameen, 2015; Dharani, K;Selvi, D, 2018).

Therefore, an STLPPs with formulation (2.2-1) is then called STLPPFI. Although, immediately every fuzzy polyhedral set $\tilde{\pi}$ formulation (2.2-1) should be converted to stochastic/default/known polyhedral set π formulation (2.2-2) via alpha-cut technique, so after converting formulation (2.2-1) to formulation (2.2-1) then converts STLPPFI to stochastic transportation linear programming problem with certainty known information on probability distribution space or STLPP.

2.3. The Literature Review

This section specific for some previous works that closes to current area paper title and around of it that helps to understand STLPPFI.

2.3.1. Stochastic Linear Programming Problem with Fuzzy Linear Partial Information on Probability Distribution (SLPPFI)

One of the related subjects and close to STLPPFI from stochastically part direction and definition on probability distribution space is first work by (Abdelaziz, F Ben;Masri, Hatem, 2005) was presented on SLPPFI. The problem was formed as:

$$\begin{gathered} Min.\ c^T(\omega)x \\ s.t.\ T(\omega)x - h(\omega) \geq 0 \\ x \in X \end{gathered}$$

Formulation 2.3-1. The SLPPFI Mathematical Formulation

where $c(\omega), T(\omega)$ and $h(\omega)$ were $(n, 1), (m, n)$ and $(m, 1)$ random vector of objective coefficients, and random matrix with random vector of constraint parameters respectively. Where SLPPFI is defined on some probability distribution space $(\Omega, 2^{\Omega}, P)$ with $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\}$ is a finite set of possible states of nature, 2^{Ω} was power set of Ω, and P is vector of probabilities $p_i = P(\{\omega = \omega_i\})$. The set X is a polyhedral set of feasible solution set that includes deterministic constraints of SLPPFI problem on probability distribution space $(\Omega, 2^{\Omega}, P)$. Although, in current paper same as this work the uncertainty unknown of information on probability distribution was with in two ways which are stochastic and fuzzy polyhedral set. Finally, the SLPPFI solution process started by deffuzzifier fuzziness on stochastic linear programming with fuzzy linear partial information on probability distribution (SLPF), then via using chance constrained approach (CCA) of stochastic transformation to determine stochastically of original problem, then minimizing expected value of random objective function coefficients on π, and using RA approach after getting the solution when the deviation or shortage had been obtained in result (Abdelaziz, F Ben;Masri, Hatem, 2005).

2.3.2. Multi-Objective Stochastic Linear Programming Problems (MSLPP)

MSLPPs is also related subjects and close to STLPPFI from stochastically part direction and related with unique-objective stochastic linear programming problems (SLPP), since MSLPP need to be convert to SLPP via multi-objective technique transformation. The relationship starts after transforming MSLPP to SLPP and also when SLPPFI to SLPP. Depending on researchers as Ben Abdelaziz and Masri are signs MSLPPs expression in some scientific research papers in 2005a, 2005b, 2010, 2012 (Abdelaziz, F Ben;Masri, Hatem, 2005; Abdelaziz, Fouad Ben;Masri, Hatem, 2010; Abdelaziz, 2012; Ben Abdelaziz, F;Masmoudi, Meryem, 2012) as well as Aouni et al at 2005 (Aouni, Belaïd;Abdelaziz, Foued Ben;Martel, Jean-Marc, 2005). So, whom were formulate SLPP as follows:

The stochastic multi-objective functions:

$$Max\ z(\omega,x)=\sum_{j=1}^{n_0} x_j c_j(\omega)$$

$$Min\ z(\omega,x)=\sum_{j=n_0+1}^{n} x_j c_j(\omega)$$

subject to both first-part and second-part stochastic constraints with domain conditions:

$$\sum_{j=1}^{n} x_j\, a_{ij}(\omega)\le b_i(\omega)\ ;i=1,2,\dots,m_0$$

$$\sum_{j=1}^{n} x_j\, a_{ij}(\omega)\ge b_i(\omega)\ ;i=m_0+1,m_0+2,\dots,m$$

$$x_j\ge 0,\forall j=1,2,\dots,n;\ x\in D,\omega\in \Omega$$

Formulation 2.3-2. The MSLPP Mathematical Formulation

where $c_j(\omega),a_{ij}(\omega),b_i(\omega)$ are non-crisp and appears stochastically, also $c_j(\omega),a_{ij}(\omega)$ and $b_i(\omega)$ are $(1,n),(m,n)$ and $(m,1)$ random matrices respectively, defined in terms of some probability space $(\Omega,2^{\Omega},P)$ with $\Omega=\{\omega_i\},i=1,2,\dots,m;j=1,2,\dots,n$ is a discrete set of events. Where x is an n-dimensional vector, and all variables need to be non-negative and all stochastic parameters need to be belonged to probability distribution space, this condition are known by conditions of restrictions in the sign of variables or feasible region conditions or domain conditions (Ameen, 2015).

2.3.3. Multi-Objective Fuzzy Stochastic Linear Programming Problems (MFSLPP)

The MFSLPP, fuzzy stochastic linear programming problems (FSLPP) and multi-objective fuzzy stochastic linear programming problems with fuzzy information on probability distribution space (MFSLPPFI) are work by (Ameen, 2015). They are also related with SLPP, since all of them should be convert to SLPP via fuzzy transformation on probability distribution space, fuzzy transformation of problem formulation, multi-objective technique transformation and stochastic transformation of problem formulation. Also, all of them are related subjects and close to STLPPFI from stochastically part direction and definition of information on probability distribution space. The relationship starts with STLPPFI after transforming MFSLPP/MFSLPPFI/FSLPP to SLPP and also when STLPPFI to STLPP then to DTLPP. Depending on researchers as Abdulqader Ameen (Ameen, 2015) are signs MFSLPPFI problems expression as follows:

The multi-objective fuzzy stochastic objective functions:

$$Max\ \tilde{z}_i(\omega,x)=\sum_{j=1}^{n}\tilde{c}_{ij}(\omega)\,x_j\ ;i=1,2,\dots,r$$

$$Min\ \tilde{z}_i(\omega,x)=\sum_{j=1}^{n}\tilde{c}_{ij}(\omega)\,x_j\ ;i=r+1,r+2,\dots,s$$

subject to both first-part and second-part fuzzy stochastic constraints:

$$\sum_{j=1}^{n} x_j\,\tilde{a}_{ij}(\omega)\le \tilde{b}_i(\omega)\ ;i=1,2,\dots,m_0$$

$$\sum_{j=1}^{n} x_j\,\tilde{a}_{ij}(\omega)\ge \tilde{b}_i(\omega)\ ;i=m_0+1,m_0+2,\dots,m$$

domain conditions:

$$x_j\ge 0,\forall j=1,2,\dots,n;$$

$x \in D, \omega \in \Omega; \{\tilde{a}_{ij}(\omega), \tilde{b}_i(\omega), \tilde{c}_{ij}(\omega)\} \in F(\mathbb{R})(\omega)$

Formulation 2.3-3. The Mathematical Formulation of MFSLPPFI model problems

where $\tilde{a}_{ij}(\omega), \tilde{b}_i(\omega)$ and $\tilde{c}_{ij}(\omega)$ are non-crisp and appears fuzzy stochastically, also $\tilde{a}_{ij}(\omega), \tilde{b}_i(\omega)$ and $\tilde{c}_{ij}(\omega)$ are $(m,n), (m,1)$ and (s,n) fuzzy random matrices respectively. It defined in terms of some probability space $(\Omega, 2^{\Omega}, P)$, with $\Omega = \{\omega_i\}, i = 1,2,\dots,s; j = 1,2,\dots,n$ is a discrete set of events and $\tilde{a}_{ij}(\omega), \tilde{b}_i(\omega), \tilde{c}_{ij}(\omega) \in F(\mathbb{R})(\omega), \tilde{a}_{ij}(\omega) = (a^L{}_{ij}, a^U{}_{ij}, \alpha_{ij}, \beta_{ij})(\omega), \tilde{b}_i(\omega) = (b^L{}_i, b^U{}_i, \alpha_i, \beta_i)(\omega)$ and $\tilde{c}_{ij}(\omega) = (c^L{}_{ij}, c^U{}_{ij}, \chi_{ij}, \psi_{ij})(\omega)$. x is an n-dimensional vector (Ameen, 2015).

3. Introduction to Problem Mathematical Formulation

This section will be discussing mathematically the organization of STLPPFI formulation problem, standard DTLPP formulation problem and data information tables of both STLPPFI and DTLPP model problems as follows:

3.1. The STLPPFI Model Problem Mathematical Formulation

The mathematical formulation of an STLPPFI con be written as follows:

The stochastic unique-objective function:

$$Min\ z(\omega, x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}(\omega)$$

subject to both deterministic availability and requirement constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i\ ; i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ; j = 1,2,\dots,n$$

with satisfying deterministic balance condition:

$$x_{ij} \geq 0, \forall i,j; i = 1,2,\dots,m; j = 1,2,\dots,n;$$
$$x \in X, \omega \in \Omega$$

Formulation 3.1-1. The Mathematical Formulation of STLPPFI

where stochastic unique-objective function is optimality condition of LPP, and it is subject to both of deterministic availability constraints and deterministic requirement constraints respectively, as well as satisfying deterministic balance condition and domain condition, where constraints are feasible solution region condition of LPP, and deterministic balance condition mean that total availability constraints satisfy total requirements constraints, and domain condition mean that non-negativity of unknown variable set and belongness of stochastic parameters to probability distribution space. Where a_i and b_j are $(m,1)$ known vector production/availability values of electricity in Kw/h and $(1,n)$ known vector demand/requirement values of using electricity in Kw/h respectively as shows in Table (3.1-1). Each of a_i, b_j are crisp which is meant does not appear scholastically. Although, $c_{ij}(\omega)$ is probably estimated cost values of transporting electricity in IQD/Kw, it is not crisp and appears scholastically should be determined it via both main transformations, and $c_{ij}(\omega)$ is (m,n) random matrix as shows in Table (3.1-1), as well as the probably estimated cost values for each $c_{ij}(\omega)$ shows in Table (3.1-2) respectively. Further, x_{ij} is (m,n) unknown matrix should be found it via suitable methods as MMCM, and it is amount of transporting electricity in Kw/h. So, the formulation (3.1-1) has fuzzy uncertainty unknown information on probability distribution. Depends on (Abdelaziz, Fouad Ben;Aouni, Belaid;El Fayedh, Rimeh, 2007; Abdelaziz, Fouad Ben;Masri, Hatem, 2010; Ameen, 2015) formulation (3.1-1) could be defined in terms of some probability distribution space $(\Omega, 2^{\Omega}, P)$, where $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\} = \{\omega_k\}, k = 1,2,\dots,N$ is a discrete set of events or a finite set of possible states of nature, 2^{Ω} is power set of Ω, and P is in terms of fuzzy uncertainty information, that assigns to each $A \in 2^{\Omega}$ the probability of occurrence $P(A)$ (i.e., P is the (s, N) matrix of probabilities $p_i = P(\{\omega = \omega_i\}), i = 1,2,\dots,s, p_i \in \tilde{\pi}, \forall i$). Although, the set X is a known polyhedral set of feasible solutions that includes deterministic constraints of STLPPFI problem.

Now, to solve formulation (3.1-1), we have to convert it into DTLPP formulation (3.2-1), then finding the set of non-negatives $x_{ij}, \forall i,j$ that minimize objective polynomial function's, satisfy constraint conditions, balanced condition and domain condition. Where the data is illustrated in Table (3.1-1) of STLPPFI formulation (3.1-1). Note that data in Table (3.1-1) may be vaguely or inaccurate values since there might be fuzzy information on probability distribution or there is not any previous information on probability distribution, so information and data will be distributed as follows:

Suppose m electricity power production stations names $G_1, G_2, G_3, \ldots$, and G_m with n cities need to be supplied with electricity names $K_1, K_2, K_3, \ldots$, and K_n as following balanced STLPPFI problem table:

Table 3.1-1. The STLPPFI Data Distribution

Power Plants	Cites				Supply Million Kw/h
	K_1	K_2	…	K_n	
G_1	$c_{11}(\omega)$	$c_{12}(\omega)$	…	$c_{1n}(\omega)$	a_1
G_2	$c_{21}(\omega)$	$c_{22}(\omega)$	…	$c_{2n}(\omega)$	a_2
⋮	⋮	⋮	⋱	⋮	⋮
G_m	$c_{m1}(\omega)$	$c_{m2}(\omega)$	…	$c_{mn}(\omega)$	a_m
Demand Million Kw/h	b_1	b_2	…	b_n	Total = (v) M Kw/h Balanced

the prices of transporting costs are stochastically, and probably estimated cost values of random matrix of $c_{ij}(\omega)$ will be as follows:

Table 3.1-2. The Probably Estimated Cost Values of P

ω	ω_1	ω_2	…	ω_s
$c_{11}(\omega)$	$c_{11}(\omega_1)$ IQD	$c_{11}(\omega_2)$ IQD	…	$c_{11}(\omega_s)$ IQD
$c_{12}(\omega)$	$c_{12}(\omega_1)$ IQD	$c_{12}(\omega_2)$ IQD	…	$c_{12}(\omega_s)$ IQD
⋮	⋮	⋮	⋱	⋮
$c_{1n}(\omega)$	$c_{1n}(\omega_1)$ IQD	$c_{1n}(\omega_2)$ IQD	…	$c_{1n}(\omega_s)$ IQD
$c_{21}(\omega)$	$c_{21}(\omega_1)$ IQD	$c_{21}(\omega_2)$ IQD	…	$c_{21}(\omega_s)$ IQD
$c_{22}(\omega)$	$c_{22}(\omega_1)$ IQD	$c_{22}(\omega_2)$ IQD	…	$c_{22}(\omega_s)$ IQD
⋮	⋮	⋮	⋱	⋮
$c_{2n}(\omega)$	$c_{2n}(\omega_1)$ IQD	$c_{2n}(\omega_2)$ IQD	…	$c_{2n}(\omega_s)$ IQD
⋮	⋮	⋮	⋱	⋮
$c_{m1}(\omega)$	$c_{m1}(\omega_1)$ IQD	$c_{m1}(\omega_2)$ IQD	…	$c_{m1}(\omega_s)$ IQD
$c_{m2}(\omega)$	$c_{m2}(\omega_1)$ IQD	$c_{m2}(\omega_2)$ IQD	…	$c_{m2}(\omega_s)$ IQD
⋮	⋮	⋮	⋱	⋮
$c_{mn}(\omega)$	$c_{mn}(\omega_1)$ IQD	$c_{mn}(\omega_2)$ IQD	…	$c_{mn}(\omega_s)$ IQD

where information of response cities on electricity power plants are fuzzy distributed (i.e., the information probability distribution shown as fuzzy polyhedral set $\tilde{\pi}$ formulation (2.2-1)). The STLPPFI formulation (3.1-1) has the stochastically expressed in its objective function coefficients, and it has fuzzily expressed in its information probability distribution space $(\Omega, 2^{\Omega}, P)$.

3.2. The DTLPP Model Problem Mathematical Formulation

The STLPPFI was discussed in previous subsection. Now, DTLPP mathematical formulation necessary to be explain, since after defuzzifying fuzziness of information on probability distribution space of STLPPFI and derandomizing stochastic/randomness of problem formulation of it, immediately STLPPFI converts to DTLPP. In addition, DTLPP can be shown as follows:

The deterministic unique-objective function:

$$Min\ z(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}$$

subject to both deterministic availability and requirement constraints with satisfying deterministic balance condition that stated before in STLPPFI formulation (3.1-1), since the remaining constraints and condition remain the same as mentioned above.

with satisfying domain condition:

$$x_{ij} \geq 0, \forall i, j; i = 1,2, \ldots, m; j = 1,2, \ldots, n; x \in X$$

Formulation 3.2-1. The Mathematical Formulation of DTLPP

where deterministic unique-objective function is optimality condition of LPP, and it is subject to both deterministic availability constraints and deterministic requirement constraints respectively, as well as satisfying deterministic balance condition and domain condition, where constraints are feasible solution region condition of LPP, and deterministic balance condition mean that total availability constraints satisfy total requirements constraints, and domain condition mean that non-negativity of unknown variable set. Where c_{ij}, a_i and b_j are (m, n) known matrix of deterministic cost values of transporting electricity in IQD/Kw, $(m, 1)$ known vector production/availability values of electricity in Kw/h, $(1, n)$ known vector demand/requirement values of using electricity in Kw/h respectively as shows in Table (3.2-1), where all of them are crisp/deterministic and does not appear scholastically. Although, x_{ij} is (m, n) unknown matrix should be found it by suitable methods as

MMCM and it is amount of transporting electricity in Kw/h. Further, the set X is a polyhedral set of feasible solutions that includes deterministic constraints of DTLPP problem.

Now, to solve formulation (3.2-1), we have to find set of non-negatives $x_{ij}, \forall i, j$ that minimize objective function, satisfy constraint conditions, balanced condition and domain condition. Where data distributed in Table (3.2-1) of DTLPP formulation (3.2-1). Note that the data in Table (3.2-1) are most approximately equivalent values to exact values, since fuzzy information on probability distribution space converts to trust/known information now. So, the information distributes as follows:

Suppose m electricity power product stations named $G_1, G_2, G_3, \ldots,$ and G_m with n cities need to be supplied with electricity names $K_1, K_2, K_3, \ldots,$ and K_n as following balanced DTLPP table:

Table 3.2-1. The DTLPP Data Distribution

Power Plants	Cites				Supply Million Kw/h
	K_1	K_2	…	K_n	
$\boldsymbol{G_1}$	c_{11}	c_{12}	…	c_{1n}	a_1
$\boldsymbol{G_2}$	c_{21}	c_{22}	…	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$\boldsymbol{G_m}$	c_{m1}	c_{m2}	…	c_{mn}	a_m
Demand Million Kw/h	b_1	b_2	…	b_n	Total = (v) M Kw/h Balanced

3.3. The Algorithm Program Outline of STLPPFI

The algorithm program outline of STLPPFI will be state in appendix section in detail.

4. Techniques for Solving STLPPFI Model Problems

This section describes those techniques are uses for solving STLPPFI till obtaining DTLPP. The solution process contains several stages. The first stage is fuzzy transformation on probability distribution space via alpha-cut technique applies, which is defuzzification of the fuzzy expression on the probability distribution space of the original STLPPFI problem and convert it to its corresponding equivalent to STLPP by creating bounded interval with unlimited possible known values from unknown probably/stochastic values. The second stage is using truth degrees technique for creating fuzzy probability subintervals from obtained interval, then LFMFs will be found from fuzzy truth degrees subintervals. After that, sketching LFMFs in one combinational figure for separation various different fuzzy regions. Then T_pFN and T_rFN creates from LFMFs, and then fuzzy truth degrees regions that showed as T_pFN and T_rFN will be defuzzified to stochastic truth degrees regions via LFRF to get finite discrete determined value set. Then obtained cases from determined value set are analyzed via testing second condition of alpha-cut technique polyhedral set to get acceptances' vector(s) for preparation to stochastic transformation. Then the third stage starts via stochastic transformation of formulation problem, which is derandomization of probably randomness value set of random variables towards its corresponding equivalent deterministic values, where stochastic transformation of objective function via EWS technique applies to transforming STLPP to DTLPP. Then the fourth stage contains solving resulted DTLPP via MMCM to getting BFS. Followed by finding optimality solution via MODI, where the optimal solution of DTLPP model problems is have minimum objective function (i.e., have minimum objective total transporting costs). Finally, selecting post optimal solution as final result via DM deciding commands which resulting solution is suitable with DM situations, which is a test of the post optimality based on the previous experience of the DM.

4.1. Fuzzy Transformation on Probability Distribution via Alpha-Cut Technique

The first transforming on an STLPPFI (i.e., formulation (3.1-1) with (2.2-1)) to STLPP (i.e., formulation (3.1-1) with (2.2-2)) is the transform of probability distribution space $(\Omega, 2^{\Omega}, P)$ from fuzzy uncertainty information that generated by fuzzy/inexact inequalities on $\tilde{\pi}$ and non-crisp of P to stochastic inequalities on π and crisp of P (i.e., P converts from formulation (2.2-1) fuzzy polyhedral set $\tilde{\pi}$ to formulation (2.2-2) stochastic polyhedral set π via using alpha-cut technique (Abdelaziz, 2012; Abdelaziz, F Ben;Masri, Hatem, 2005; Ameen, 2015). In general alpha-cut technique work on polyhedral sets to convert them from probably estimated unknown values to bounded interval with unlimited possible known values to obtain STLPP. Note that LFMF of singleton T_pFN or T_pFN are simple and contain only one LFMF for one fuzzy number, but LFMF of fuzzy polyhedral set are complex for example in current fuzzy truth degrees set contain seven LFMFs for each p_i where first LFMF has T_pFN, second one has T_rFN and so on.

Now, all fuzzy inequalities $\sum_{j=1}^{n} a_{ij}p_j \lesssim b_i, i = 1, \dots, s$ of fuzzy polyhedral set formulation (2.2-1) could be shown as membership function $\mu_i, i = 1, \dots, s$ for T_pFN and T_rFN as following two LFMF's:

$$\mu_i(p) = \begin{cases} \dfrac{\sum_{j=1}^{N} a_{ij}p_j - (-b_i + d_i)}{2(b_i - d_i)} & -b_i + d_i \le \sum_{j=1}^{N} a_{ij}p_j \le b_i - d_i \\ 1 & b_i - d_i \le \sum_{j=1}^{N} a_{ij}p_j \le b_i \\ \dfrac{(b_i + d_i) - \sum_{j=1}^{N} a_{ij}p_j}{d_i} & b_i \le \sum_{j=1}^{N} a_{ij}p_j \le b_i + d_i \\ 0 & \sum_{j=1}^{N} a_{ij}p_j \ge b_i + d_i \end{cases}$$

Formulation 4.1-1. The T_pFN Linear Fuzzy Membership Function (PLFMF) of Fuzzy Polyhedral Set

$$\mu_i(p) = \begin{cases} 1 & \sum_{j=1}^{N} a_{ij}p_j \le b_i \\ \dfrac{(b_i + d_i) - \sum_{j=1}^{N} a_{ij}p_j}{d_i} & b_i \le \sum_{j=1}^{N} a_{ij}p_j \le b_i + d_i \\ 0 & \sum_{j=1}^{N} a_{ij}p_j \ge b_i + d_i \end{cases}$$

Formulation 4.1-2. The T_rFN Linear Fuzzy Membership Function (RLFMF) of Fuzzy Polyhedral Set

where $p = (p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N$ is a probability distribution space vector, $d = (d_1, d_2, \dots, d_N)$ is a vagueness level vector and it is used to each value of p exceeding $b + d$ should be neglected, alpha-cut level vector are $(\alpha_1, \alpha_2, \dots, \alpha_N)$ and the credibility degree of DM about information on probability distribution of $p_1, p_2, \dots, p_N$ are around determinated values respectively i.e., $P(p_1) \approx b_1, P(p_2) \approx b_2, \dots, and\ P(p_N) \approx b_N$ then we get $\big(P(p_1), P(p_2), \dots, P(p_N)\big) \approx (b_1, b_2, \dots, b_N)$ (Ameen, 2015).

The following alpha-cut technique formula will be applied for each fuzzy inequality in formulation (2.2-1) as follows:

$$\pi^k = \left\{ \begin{array}{c} p = (p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N; \\ b_k - d_k(1 - \alpha_k) \le p_k \le b_k + d_k(1 - \alpha_k); \\ \sum_{k=1}^{N} p_k = 1, \forall p_k \ge 0, k = 1, \dots, N \end{array} \right\}$$

Formulation 4.1-3. The Alpha-Cut Technique Formula

Now, it can be applying the truth degrees technique on probability distribution space, then analyzing cases, and finally applying stochastic transformation of objective function via EWS technique (Ameen, 2015).

4.2. The Truth Degrees Technique on Probability Distribution Space

In this subsection, we focus on probability distribution space more deeply, after defuzzifying fuzzy uncertainty information on probability distribution and it converts to certainty known information on probability distribution. The obtained probability intervals could be defined as a fuzzy set of the truth degrees technique on probability distribution space, where truth degrees technique improves from fuzzy set of optimistic and pessimistic in probability distribution space of researcher (Ameen, 2015). In addition, we classified continuous interval of fuzzy probability distribution polyhedral set to ten equal subintervals or eleven elementary equal distance points each from each other, then seven important/efficient points from it are given via defuzzifying those eleven points by LFRF formulation (4.2-5) and formulation (4.2-6) with two boundary points then get nine important/efficient points and mistake other unnecessary points (i.e., LFRF technique algorithm uses as a part of truth degrees technique algorithm). Note that the obtained nine important/efficient points from defuzzifying are different from first elementary eleven points since those eleven points in first time are not necessarily to be effective points but surely the obtained points are efficient points. This technique obtains from logic fuzziness of human normal languages via degrees value of truth in numeric logical answering of a question. For example, when anyone is asked to give opinions on the expected result of a random subject, phenomenon, job, routine or healthy status now then answers of it will be fuzzy logic values, as this question (Are you fine?) the answer set is {1.0 Yes/Perfect, 0.9 Excellent, 0.8 Very Good, 0.7 Good, 0.6 Well, 0.5 Moderate, 0.4 Some, 0.3 Somewhat, 0.2 Little, 0.1 Very Little, 0 No/Bad}. In general, the truth degrees technique on probability distribution space uses for dividing continuous intervals to deterministic discrete values set of important/efficient points. So, the truth degree variable values x contain n terms $x_1, x_2, \dots, x_n$ and the series of those terms are $\{x_1, x_2, \dots, x_n\}$ (Ameen, 2015; Guo, Haiying;Wang, Xiaosheng;Zhou, Shaoling, 2015; Abdelaziz, 2012). Note that the number of terms starts with $\{3,7,10,14, \dots, x_{n-1}+3, x_n+4\}$ that causes to obtain T_rFN as first place then T_pFN as second place and so on. Then splitting each continuous interval of the default probability distribution π in formulation (2.2-2) into ten continuous subintervals as shown as follows:

$$p = (p_1, p_2, \dots, p_N)^T \in \mathbb{R}^N;$$
$$\sum_{i=1}^{N} p_i = 1, \forall p_i \geq 0, \alpha \leq p_i \leq \beta, i = 1, \dots, N$$

where $\alpha, \beta, \varphi \in \mathbb{R}$, with $\varphi = a_5 = b_5$, and

$$[\alpha, \beta] = [\alpha, a_1] \cup [a_1, a_2] \cup [a_2, a_3] \cup [a_3, a_4] \cup [a_4, \varphi] \cup [\varphi, b_4] \cup [b_4, b_3] \cup [b_3, b_2] \cup [b_2, b_1] \cup [b_1, \beta]$$

Formulation 4.2-1. The Splitting Original Interval of Probability Distribution Space

such that those ten continuous subintervals classified into seven stochastic truth degrees regions:

$$A = \{\{\alpha, a_2\}, \{a_1, \varphi\}, \{a_2, \varphi\}, \{a_3, b_3\}, \{\varphi, b_2\}, \{\varphi, b_1\}, \{b_2, \beta\}\}$$
$$A = \begin{Bmatrix} Little, Some, Moderate, Well, \\ Good, Very\ Good, Excellent \end{Bmatrix}$$
$$A = \{Li, So, Mo, We, Go, Ve, Ex\};\ \forall p_i \in \pi, i = 1, \dots, N$$

Formulation 4.2-2. Stochastic Truth Degrees Regions Set

where $Li, So, Mo, We, Go, Ve\ and\ Ex$ are stochastic of little, some, moderate, well, good, very good and excellent respectively, and also $[\alpha, \varphi]$ and $[\varphi, \beta]$ are fail region and pass region probability distribution respectively, and $\varphi = a_5 = b_5$, as shown in Figure (4.2-1).

when $p_i \in \tilde{\pi}$ then we obtain fuzzy truth degrees set of p_i that could be shown as:

$$\tilde{A} = \{\{\widetilde{\alpha, a_2}\}, \{\widetilde{a_1, \varphi}\}, \{\widetilde{a_2, \varphi}\}, \{\widetilde{a_3, b_3}\}, \{\widetilde{\varphi, b_2}\}, \{\widetilde{\varphi, b_1}\}, \{\widetilde{b_2, \beta}\}\}$$
$$= \begin{Bmatrix} Fuzzy\ Little, Fuzzy\ Some, Fuzzy\ Moderate, Fuzzy\ Well, \\ Fuzzy\ Good, Fuzzy\ Very\ Good, Fuzzy\ Excellent \end{Bmatrix}$$
$$= \{\widetilde{Li}, \widetilde{So}, \widetilde{Mo}, \widetilde{We}, \widetilde{Go}, \widetilde{Ve}, \widetilde{Ex}\};\ \forall p_i \in \tilde{\pi}, i = 1, \dots, N$$

Formulation 4.2-3. Fuzzy Truth Degrees Regions Set

Note that fuzzy truth degrees set formulation (4.2-3) should be convert to stochastic truth degrees set formulation (4.2-2) via LFRF formulation (4.2-5) and formulation (4.2-6).

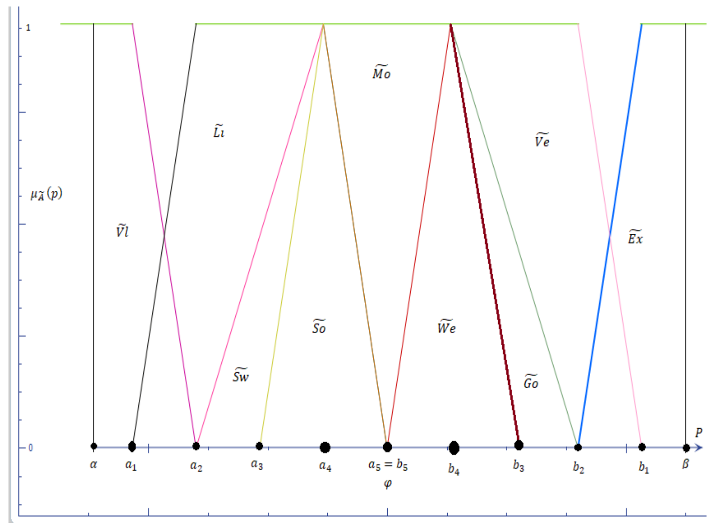

Figure 4.2-1. Fuzzy Truth Degree Regions Set of Probability Distribution Space

The $\mu_k(p_i), k = 1,2,..,7; i = 1,2, \dots, N$ are LFMFs for entire both types T_pFN and T_rFN regions of fuzzy truth degrees set $\tilde{A} = \{\widetilde{Li}, \widetilde{So}, \widetilde{Mo}, \widetilde{We}, \widetilde{Go}, \widetilde{Ve}, \widetilde{Ex}\}$ of each p_i respectively on information of probability distribution that defined in T_pFN and T_rFN (Abdelaziz, F Ben;Masri, Hatem, 2005; Abdelaziz, Fouad Ben;Masri, Hatem, 2009; Ameen, 2015) were obtained from both formulation (4.1-1) and formulation (4.1-2) will be as follows:

$$\mu_1(p_i) = \begin{cases} 1 & \alpha \leq p_i \leq a_1 \\ \dfrac{a_2 - P}{a_2 - a_1} & a_1 \leq p_i \leq a_2, \\ 0 & otherwise \end{cases} \qquad \mu_2(p_i) = \begin{cases} \dfrac{P - a_1}{a_2 - a_1} & a_1 \leq p_i \leq a_2 \\ 1 & a_2 \leq p_i \leq a_4 \\ \dfrac{\varphi - P}{\varphi - a_4} & a_4 \leq p_i \leq \varphi \\ 0 & otherwise \end{cases}$$

$$\mu_3(p_i)=\begin{cases}\frac{P-a_2}{a_4-a_2} & a_2\le p_i\le a_4\\ 1 & p_i=a_4\\ \frac{\varphi-P}{\varphi-a_4} & a_4\le p_i\le\varphi\\ 0 & otherwise\end{cases},\quad \mu_4(p_i)=\begin{cases}\frac{P-a_3}{a_4-a_3} & a_3\le p_i\le a_4\\ 1 & a_4\le p_i\le b_4\\ \frac{b_3-P}{b_3-b_4} & b_4\le p_i\le b_3\\ 0 & otherwise\end{cases}$$

$$\mu_5(p_i)=\begin{cases}\frac{P-\varphi}{b_4-\varphi} & \varphi\le p_i\le b_4\\ 1 & p_i=b_4\\ \frac{b_2-P}{b_2-b_4} & b_4\le p_i\le b_2\\ 0 & otherwise\end{cases},\quad \mu_6(p_i)=\begin{cases}\frac{P-\varphi}{b_4-\varphi} & \varphi\le p_i\le b_4\\ 1 & b_4\le p_i\le b_2\\ \frac{b_1-P}{b_1-b_2} & b_2\le p_i\le b_1\\ 0 & otherwise\end{cases}$$

$$\mu_7(p_i)=\begin{cases}\frac{P-b_2}{b_1-b_2} & b_2\le p_i\le b_1\\ 1 & b_1\le p_i\le\beta\\ 0 & otherwise\end{cases}$$

Formulation 4.2-4. The $\mu_k(p_i)$ Linear Fuzzy Membership Functions

since LFRF uses as a technical tool of fuzzy transformation of problem formulation, as well as it uses also as particular step process of the truth degrees technique on probability distribution to deffuzzifier fuzzy truth degrees intervals to stochastic truth degrees intervals, then via LFRF technique finite deterministic bounded discreate values set are obtained. In addition, to achieve fuzzy transformation, LFRF to defuzzify fuzzy coefficients and parameters in fuzzy programing problems (FPP) is used. So, one of most useful ranking functions proposed by various researchers is (Mahdavi-Amiri, N;Nasseri, SH, 2006; Mahdavi-Amiri;NezamNasseri;Seyed Hadi, 2007), since it could use for entire types of T_pFN and T_rFN because immediately after transforming fuzzy numbers to real number, the value of it will remains in the same interval and could be shown as average of fuzzy number components (Ameen, 2015). Therefore, LFRF uses in converting intervals of truth degrees to important/efficient points.

Now, for the T_pFN $\widetilde{a}=(a^L,a^U,\alpha,\beta)$, we could use formulation $R(F)(\widetilde{a})$ as following:

$$R(F)(\widetilde{a})=\frac{1}{2}\int_0^1\left(\inf(\widetilde{a_\lambda})+sup(\widetilde{a_\lambda})\right)d\lambda=\frac{a^L+a^U}{2}+\frac{\beta-\alpha}{4}$$

Formulation 4.2-5. The Linear Fuzzy Ranking Function Technique Formula For T_pFN

Also, for the T_rFN $\widetilde{a}=(a,\alpha,\beta)$, we could use formulation $R(F)(\widetilde{a})$ as following:

$$R(F)(\widetilde{a})=\frac{1}{2}\int_0^1\left(\inf(\widetilde{a_\lambda})+sup(\widetilde{a_\lambda})\right)d\lambda=a+\frac{\beta-\alpha}{4}$$

Formulation 4.2-6. The Linear Fuzzy Ranking Function Technique Formula For T_rFN

Thus, by using above LFRF technique formulation (4.2-5) and formulation (4.2-6) defuzzified those $\mu_k(p_i), k=1,2,..,7; i=1,2,\dots,N$ LFMFs of formulation (4.2-3) and becomes to deterministic discrete crisp values for each p_i of $p=(p_1,p_2,\dots,p_N)^T\in\mathbb{R}^N$.

4.2.1. Analyzing Cases

Analyzing cases starts after all p_i of $p=(p_1,p_2,\dots,p_N)^T\in\mathbb{R}^N$ of probability distribution space vector converts to crisp values, and then each case that does not apply $\sum_{k=1}^{N}p_k=1, p_k\ge 0, k=1,\dots,N$ of formulation (4.1-3) should be neglected to get acceptance vector to preparing to stochastic transformation.

4.3. Stochastic Transformation of Objective Function via Expectation Weighted Summation Technique

The stochastic transformation of objective functions of formulation (3.1-1) via EWS technique on random objective coefficients applies to convert STLPP to DTLPP, for a probability distribution space $p=(p_1,p_2,\dots,p_{\mathrm{N}})^T\in\mathbb{R}^{\mathrm{N}}$, then stochastic transformation of objective coefficients is applied as follows:

$$\begin{aligned}Min\ Exp_{P\in\pi}\,z(\omega,x)&=Min\ Exp_{P\in\pi}\sum_{i=1}^{\mathrm{m}}\sum_{j=1}^{\mathrm{n}}c_{ij}(\omega)x_{ij}\\&=Min\sum_{i=1}^{\mathrm{m}}\sum_{j=1}^{\mathrm{n}}x_{ij}\left(Exp_{P\in\pi}\,c_{ij}(\omega)\right)\\&=Min\sum_{i=1}^{\mathrm{m}}\sum_{j=1}^{\mathrm{n}}x_{ij}\left(\sum_{k=1}^{\mathrm{N}}c_{ij}(\omega_k)p_k\right)\\&=Min\sum_{i=1}^{\mathrm{m}}\sum_{j=1}^{\mathrm{n}}x_{ij}\,c_{ij}=Min\ z(x)\end{aligned}$$

$$\forall k\in\mathbb{N}; i=1,2,\dots,m; j=1,2,\dots,n, k=1,2,\dots,N$$

Or

$$Exp_{P\in\pi}\, c_{ij}(\omega) = \sum_{k=1}^{N} c_{ij}(\omega_k)p_k = c_{ij}$$

$$\forall k \in \mathbb{N}; i = 1,2,\dots,m\,; j = 1,2,\dots,n, k = 1,2,\dots,N$$

Formulation 4.3-1. The Expectation Weighted Summation Technique Procedure

4.4. Finding Basic Feasible Solution for DTLPP via Matrix Minima Cost Method

When STLPPFI converts to DTLPP, basic feasible solution (BFS) can be found for it via various methods as north-west corner method (NWCM), row minima cost method (RMCM), column minima cost method (CMCM), MMCM and Vogel approximation method (VAM). The MMCM are focused on it, and will be illustrated in solving problem example in next section (Reeb, James Edmund;Leavengood, Scott A, 2002; Winston, Wayne L;Goldberg, Jeffrey B, 2004; Sharma, 1974; Sengamalaselvi, 2017).

4.5. Finding Optimal Solution for DTLPP via Modify Distribution Method

When BFS found for DTLLP via MMCM, then stepping stone method (SSM) and modify distribution method (MODI) are had to check BFS is optimal or not, and also both methods are used to find optimal solution when BFS is not optimal. The MODI is recommended to use (Reeb, James Edmund;Leavengood, Scott A, 2002; Winston, Wayne L;Goldberg, Jeffrey B, 2004; Sharma, 1974; Sengamalaselvi, 2017).

4.6. Selecting Post Optimal Solution for DTLPP

Selecting post optimality solution as final result via DM deciding commands among entire obtained solutions that suitable with DM situations based on previous information of the DM, and his/her priority for solving the problem.

5. The Real-Life Application Problem Example

In this section, an example within the framework of the STLPPFI (formulation (3.1-1) with (2.2-1)) of section three is presented. The section deals with a real-life application problem in the stochastic electricity environment under fuzzy information of probability distribution. It shows the application of our proposed methodology in the framework of the stochastic environment to transform the STLPPFI in (formulation (3.1-1) with (2.2-1)) into DTLPP in formulation (3.2-1) problem and solving it.

- Fuzzy and stochastic transformation in the uncertainty probability distribution and the STLPPFI respectively.
- Converting STLPPFI to their DTLPP problems.
- Investigating and implementing of the solution algorithm of the MMCM, and MODI.

5.1. Illustrate Example

Suppose three electricity power production stations in Kurdistan region-Iraq are considered and named G1, G2 and G3 with four cities need to be supplied with electricity. City names are Hawler, Duhok, Sulaymani and Halabja (Government, 2020; TV, 2021) as following balanced STLPPFI problem table:

Table 5.1-1. The Data Distribution of an Electricity STLPPFI Application Problem

Power Plants	Cites HR	DK	SI	HA	Supply Million Kw/h
G_1	$c_{11}(\omega)$	$c_{12}(\omega)$	$c_{13}(\omega)$	$c_{14}(\omega)$	36
G_2	$c_{21}(\omega)$	$c_{22}(\omega)$	$c_{23}(\omega)$	$c_{24}(\omega)$	51
G_3	$c_{31}(\omega)$	$c_{32}(\omega)$	$c_{33}(\omega)$	$c_{34}(\omega)$	42
Demand Million Kw/h	46	22	31	30	Total = 129 M Kw/h Balanced

where price transporting costs are stochastically, and depend on (Government, 2020; TV, 2021) estimated cost values of random matrix of each $c_{ij}(\omega)$ will be as follows:

Table 5.1-2. The Probably Estimated Cost Values of P

ω	ω_1	ω_2	ω_3
$c_{11}(\omega)$	7.980 IQD	7.990 IQD	8.000 IQD
$c_{12}(\omega)$	6.000 IQD	5.990 IQD	5.980 IQD
$c_{13}(\omega)$	9.990 IQD	9.980 IQD	10.00 IQD
$c_{14}(\omega)$	8.880 IQD	8.890 IQD	9.000 IQD
$c_{21}(\omega)$	9.000 IQD	8.980 IQD	8.960 IQD
$c_{22}(\omega)$	11.98 IQD	11.96 IQD	12.00 IQD
$c_{23}(\omega)$	13.00 IQD	12.98 IQD	12.96 IQD
$c_{24}(\omega)$	6.980 IQD	6.960 IQD	7.000 IQD
$c_{31}(\omega)$	13.96 IQD	13.98 IQD	14.00 IQD
$c_{32}(\omega)$	9.000 IQD	8.980 IQD	8.960 IQD
$c_{33}(\omega)$	15.96 IQD	15.98 IQD	16.00 IQD
$c_{34}(\omega)$	4.990 IQD	5.000 IQD	4.980 IQD

where information of response cities on electricity power plants are fuzzy distributed as fuzzy polyhedral set $\widetilde{\pi}$ of uncertainty unknown information on probability distribution (Government, 2020; TV, 2021) as follows:

$$\widetilde{\pi} = \left\{ \begin{matrix} p = (p_1, p_2, p_3)^T \in \mathbb{R}^3; \\ Ap \preccurlyeq b; \sum_{i=1}^{3} p_i = 1\,; \forall p_i \geq 0; i = 1,2,3 \end{matrix} \right\}$$

Formulation 5.1-1. The Fuzzy Polyhedral Information Set $\widetilde{\boldsymbol{\pi}}$ of Real-Life Application Problem

5.2. The Solution Method Process

The problem could formulate as follows:

The stochastic unique-objective function:

$$Min\ z(\omega, x) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}(\omega)$$

$$= x_{11}c_{11}(\omega) + x_{12}c_{12}(\omega) + x_{13}c_{13}(\omega) + x_{14}c_{14}(\omega)$$
$$+x_{21}c_{21}(\omega) + x_{22}c_{22}(\omega) + x_{23}c_{23}(\omega) + x_{24}c_{24}(\omega)$$
$$+x_{31}c_{31}(\omega) + x_{32}c_{32}(\omega) + x_{33}c_{33}(\omega) + x_{34}c_{34}(\omega)$$

subject to both deterministic availability and requirement constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} = 36$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 51$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 42$$
$$x_{11} + x_{21} + x_{31} = 46; x_{12} + x_{22} + x_{32} = 22$$
$$x_{13} + x_{23} + x_{33} = 31; x_{14} + x_{24} + x_{34} = 30$$

with satisfying deterministic balance condition and domain condition respectively:

$$\sum_{i=1}^{3} a_i = 36 + 51 + 42 =$$
$$\sum_{j=1}^{4} b_j = 46 + 22 + 31 + 30 = 129$$
$$x_{ij} \geq 0, \forall i = 1,2,3,; \forall j = 1,2,3,4;\ x \in X, \omega \in \Omega$$

Formulation 5.2-1. The STLPPFI Mathematical Formulation of Application Problem

the problem has uncertainty expression in both randomness for objective coefficients and fuzziness for information probability distribution space $(\Omega, 2^{\Omega}, P)$. The STLPPFI solution's process starts from deffuzzifier fuzziness and derandomizing randomness of it respectively via two main transformations then solving via suitable methods as MMCM, finding optimal solution via MODI and selecting post optimality solution. Where a_i, b_j are crisp and does not appear scholastically and they are $(3,1)$ and $(1,4)$ known vectors as shows in Table (5.1-1) respectively, and c_{ij} is not crisp and appears scholastically should be determined it via suitable transformations, and $c_{ij}(\omega)$ is $(3,4)$ random matrix as shows in Table (5.1-1) as well as estimated probably cost values for each $c_{ij}(\omega)$ shows in Table (5.1-2) respectively, and x_{ij} is $(3,4)$ unknown matrix should be found it via MMCM. The formulation (5.2-1) is defined in terms of some probability distribution space $(\Omega, 2^{\Omega}, P)$, where $\Omega = \{\omega_1, \omega_2, \omega_3\} = \{\omega_i\}, i = 1,2,3$ is a discrete set of events or a finite set of possible states of nature, 2^{Ω} is power set of Ω, and P is fuzzy uncertainty unknown probability distribution space. Where that P assigns to each $A \in 2^{\Omega}$ is the probability of occurrence $P(A)$ (i.e., P is the $(12,3)$ matrix of probabilities $p_i = P(\{\omega = \omega_i\}), i = 1,2, \ldots, 12, p_i \in \widetilde{\pi}, \forall i)$, with fuzzy distribution information of response cities on electricity power plants as fuzzy polyhedral set $\widetilde{\pi}$ formulation (5.1-1). Also, the set X is a polyhedral set of feasible solutions that includes determined constraints of problem on probability distribution space $(\Omega, 2^{\Omega}, P)$. To solve formulation (5.2-1) need to find set of non-negatives $x_{ij}, \forall i, j$ that minimize objective function, satisfies constraints, balance condition and domain conditions.

Now, the solution process classified over steps/stages as follows:

5.2.1. Stage 1: Suppositions of Stochastics

Suppose that the credibility degree of DM about information on probability distribution of p_1, p_2, p_3 are around $\frac{1}{2}, \frac{1}{5}, \frac{1}{10}$ respectively, where $P(p_1) \cong b_1 = \frac{1}{2}, P(p_2) \cong b_2 = \frac{1}{5}, P(p_3) \cong b_3 = \frac{1}{10}$ which is mean $(b_1, b_2, b_3) = \left(\frac{1}{2}, \frac{1}{5}, \frac{1}{10}\right)$, since the information on probability distribution are fuzzy. So, the suppositions have been started, where the vagueness levels are $(d_1, d_2, d_3) = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$ and the alpha-cut levels are $(\alpha_1, \alpha_2, \alpha_3) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$. Note that the vagueness levels and the alpha-cut levels are controls length of interval in each p_i, which is allows to reuse both of them again k-times as it is necessary to get most close approximate values from exact values till discover which values have efficient on solution process results.

5.2.2. Stage 2: Fuzzy Transformation on Probability Distribution Space

The fuzzy transformation on probability distribution will be applied via alpha-cut technique formulation (4.1-3) for each fuzzy inequality polyhedral set formulation (5.1-1) as follows:

$$\pi^{1,2,3} = \left\{ \begin{array}{c} p = (p_1, p_2, p_3)^T \in \mathbb{R}^3; \\ b_i - d_i(1-\alpha_i) \le p_i \le b_i + d_i(1-\alpha_i); \\ \sum_{i=1}^{3} p_i = 1, \forall p_i \ge 0, i = 1,2,3 \end{array} \right\}$$

Then formulation (5.2-1) converts from STLPPFI to STLPP via applying above alpha-cut formula for each $p_{1,2,3}$, then we get:

$$b_1 - d_1(1-\alpha_1) \le p_1 \le b_1 + d_1(1-\alpha_1)$$
$$\Rightarrow \frac{1}{2} - \frac{1}{6}\left(1 - \frac{1}{2}\right) \le p_1 \le \frac{1}{2} + \frac{1}{6}\left(1 - \frac{1}{2}\right)$$
$$\frac{25}{60} \le p_1 \le \frac{35}{60}$$
$$b_2 - d_2(1-\alpha_2) \le p_2 \le b_2 + d_2(1-\alpha_2)$$
$$\Rightarrow \frac{1}{5} - \frac{1}{6}\left(1 - \frac{1}{2}\right) \le p_2 \le \frac{1}{5} + \frac{1}{6}\left(1 - \frac{1}{2}\right)$$
$$\frac{7}{60} \le p_2 \le \frac{17}{60}$$
$$b_3 - d_3(1-\alpha_3) \le p_3 \le b_3 + d_3(1-\alpha_3)$$
$$\Rightarrow \frac{1}{10} - \frac{1}{6}\left(1 - \frac{1}{2}\right) \le p_3 \le \frac{1}{10} + \frac{1}{6}\left(1 - \frac{1}{2}\right)$$
$$\frac{1}{60} \le p_3 \le \frac{11}{60}$$

Now, alpha-cut technique applied, which is fuzziness of probability distribution of STLPPFI are removed, and STLPPFI transfers to STLPP by creating bounded interval with unlimited possible known values from unknown probably/stochastic value (i.e., $b_i; \forall i = 1,2,3$ converted from stochastic values $b_{1,2,3} = \frac{1}{2}, \frac{1}{5}, \frac{1}{10}$ to bounded interval with unlimited possible known values $p_{i=1,2,3}$ as $\frac{25}{60} \le p_1 \le \frac{35}{60}$, $\frac{7}{60} \le p_2 \le \frac{17}{60}$, $\frac{1}{60} \le p_1 \le \frac{11}{60}$). Then, fuzzy polyhedral set formulation (5.1-1) of STLPPFI formulation (5.2-1) converts to stochastic polyhedral version set as follows:

$$\pi = \left\{ p = (p_1, p_2, p_3)^T \in \mathbb{R}^3; Ap \le b; \sum_{i=1}^{3} p_i = 1; \forall p_i \ge 0; i = 1,2,3 \right\}$$

Thus, the values of $p_{i=1,2,3}$ will occurs as:

$$\frac{25}{60} \le p_1 \le \frac{35}{60}, \frac{7}{60} \le p_2 \le \frac{17}{60}, \frac{1}{60} \le p_1 \le \frac{11}{60}$$

5.2.3. Stage 3: The Truth Degrees Technique on Probability Distribution Intervals

The current step will be converting each bounded interval with unlimited possible known values to bounded discrete finite possible known values set via truth degrees technique, to choose best effective values for each $p_{1,2,3}$ in each interval we use truth degrees logical values. First, we divide each interval into ten parts as follows:

$$\frac{25}{60} \le p_1 \le \frac{35}{60} \Rightarrow \left\{\left[\frac{25}{60}, \frac{35}{60}\right]\right\}$$
$$= \left\{\frac{25}{60}, \frac{26}{60}, \frac{27}{60}, \frac{28}{60}, \frac{29}{60}, \frac{30}{60}, \frac{31}{60}, \frac{32}{60}, \frac{33}{60}, \frac{34}{60}, \frac{35}{60}\right\}$$
$$\frac{7}{60} \le p_2 \le \frac{17}{60} \Rightarrow \left\{\left[\frac{7}{60}, \frac{17}{60}\right]\right\}$$
$$= \left\{\frac{7}{60}, \frac{8}{60}, \frac{9}{60}, \frac{10}{60}, \frac{11}{60}, \frac{12}{60}, \frac{13}{60}, \frac{14}{60}, \frac{15}{60}, \frac{16}{60}, \frac{17}{60}\right\}$$
$$\frac{1}{60} \le p_3 \le \frac{11}{60} \Rightarrow \left\{\left[\frac{1}{60}, \frac{11}{60}\right]\right\}$$
$$= \left\{\frac{1}{60}, \frac{2}{60}, \frac{3}{60}, \frac{4}{60}, \frac{5}{60}, \frac{6}{60}, \frac{7}{60}, \frac{8}{60}, \frac{9}{60}, \frac{10}{60}, \frac{11}{60}\right\}$$

5.2.4. Stage 4: Finding Modulus/Membership Functions to Truth Degrees Set

Formulation (4.2-3) with Figure (4.2-1) will be used to fuzzifier truth degrees set and to find seven LFMFs' for each $p_{1,2,3}$ as follows:

$$\mu_1(p_1) = \begin{cases} 1 & \frac{25}{60} \le p_1 \le \frac{26}{60} \\ \dfrac{\frac{27}{60} - P}{\frac{27}{60} - \frac{26}{60}} & \frac{26}{60} \le p_1 \le \frac{27}{60} \\ 0 & otherwise \end{cases}$$

$$\mu_2(p_1)=\begin{cases}\dfrac{P-\frac{26}{60}}{\frac{27}{60}-\frac{26}{60}};\dfrac{26}{60}\le p_1\le\dfrac{27}{60}\\ 1;\ \dfrac{27}{60}\le p_1\le\dfrac{30}{60}\\ \dfrac{\frac{30}{60}-P}{\frac{30}{60}-\frac{29}{60}};\ \dfrac{29}{60}\le p_1\le\dfrac{30}{60}\\ 0;\quad otherwise\end{cases}$$

$$\mu_3(p_1)=\begin{cases}\dfrac{P-\frac{27}{60}}{\frac{29}{60}-\frac{27}{60}};\dfrac{27}{60}\le p_1\le\dfrac{29}{60}\\ 1;p_1=\dfrac{29}{60}\\ \dfrac{\frac{30}{60}-P}{\frac{30}{60}-\frac{29}{60}};\ \dfrac{29}{60}\le p_1\le\dfrac{30}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_4(p_1)=\begin{cases}\dfrac{P-\frac{28}{60}}{\frac{29}{60}-\frac{28}{60}};\dfrac{28}{60}\le p_1\le\dfrac{29}{60}\\ 1;\ \dfrac{29}{60}\le p_1\le\dfrac{31}{60}\\ \dfrac{\frac{33}{60}-P}{\frac{33}{60}-\frac{31}{60}};\ \dfrac{31}{60}\le p_1\le\dfrac{33}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_5(p_1)=\begin{cases}\dfrac{P-\frac{30}{60}}{\frac{31}{60}-\frac{30}{60}};\dfrac{30}{60}\le p_1\le\dfrac{31}{60}\\ 1;p_1=\dfrac{31}{60}\\ \dfrac{\frac{33}{60}-P}{\frac{33}{60}-\frac{31}{60}};\ \dfrac{31}{60}\le p_1\le\dfrac{33}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_6(p_1)=\begin{cases}\dfrac{P-\frac{30}{60}}{\frac{31}{60}-\frac{30}{60}};\dfrac{30}{60}\le p_1\le\dfrac{31}{60}\\ 1;\ \dfrac{31}{60}\le p_1\le\dfrac{33}{60}\\ \dfrac{\frac{34}{60}-P}{\frac{34}{60}-\frac{33}{60}};\ \dfrac{33}{60}\le p_1\le\dfrac{34}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_7(p_1)=\begin{cases}\dfrac{P-\frac{33}{60}}{\frac{34}{60}-\frac{33}{60}};\ \dfrac{33}{60}\le p_1\le\dfrac{34}{60}\\ 1;\ \dfrac{34}{60}\le p_1\le\dfrac{35}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_1(p_2)=\begin{cases}1;\ \dfrac{7}{60}\le p_2\le\dfrac{8}{60}\\ \dfrac{\frac{9}{60}-P}{\frac{9}{60}-\frac{8}{60}};\ \dfrac{8}{60}\le p_2\le\dfrac{9}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_2(p_2)=\begin{cases}\dfrac{P-\frac{8}{60}}{\frac{9}{60}-\frac{8}{60}};\dfrac{8}{60}\le p_2\le\dfrac{9}{60}\\ 1;\ \dfrac{9}{60}\le p_2\le\dfrac{11}{60}\\ \dfrac{\frac{12}{60}-P}{\frac{12}{60}-\frac{11}{60}};\ \dfrac{11}{60}\le p_2\le\dfrac{12}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_3(p_2)=\begin{cases}\dfrac{P-\frac{9}{60}}{\frac{11}{60}-\frac{9}{60}};\dfrac{9}{60}\le p_2\le\dfrac{11}{60}\\ 1;p_2=\dfrac{11}{60}\\ \dfrac{\frac{12}{60}-P}{\frac{12}{60}-\frac{11}{60}};\ \dfrac{11}{60}\le p_2\le\dfrac{12}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_4(p_2)=\begin{cases}\dfrac{P-\frac{10}{60}}{\frac{11}{60}-\frac{10}{60}};\dfrac{10}{60}\le p_2\le\dfrac{11}{60}\\ 1;\ \dfrac{11}{60}\le p_2\le\dfrac{13}{60}\\ \dfrac{\frac{14}{60}-P}{\frac{14}{60}-\frac{13}{60}};\ \dfrac{13}{60}\le p_2\le\dfrac{14}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_5(p_2)=\begin{cases}\dfrac{P-\frac{12}{60}}{\frac{13}{60}-\frac{12}{60}};\dfrac{12}{60}\le p_2\le\dfrac{13}{60}\\ 1;p_2=\dfrac{13}{60}\\ \dfrac{\frac{15}{60}-P}{\frac{15}{60}-\frac{13}{60}};\ \dfrac{13}{60}\le p_2\le\dfrac{15}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_6(p_2)=\begin{cases}\dfrac{P-\frac{12}{60}}{\frac{13}{60}-\frac{12}{60}};\dfrac{12}{60}\le p_2\le\dfrac{13}{60}\\ 1;\ \dfrac{13}{60}\le p_2\le\dfrac{15}{60}\\ \dfrac{\frac{16}{60}-P}{\frac{16}{60}-\frac{15}{60}};\ \dfrac{15}{60}\le p_2\le\dfrac{16}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_7(p_2)=\begin{cases}\dfrac{P-\frac{15}{60}}{\frac{16}{60}-\frac{15}{60}};\ \dfrac{15}{60}\le p_2\le\dfrac{16}{60}\\ 1;\ \dfrac{16}{60}\le p_2\le\dfrac{17}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_1(p_3)=\begin{cases}1;\ \dfrac{1}{60}\le p_3\le\dfrac{2}{60}\\ \dfrac{\frac{3}{60}-P}{\frac{3}{60}-\frac{2}{60}};\ \dfrac{2}{60}\le p_3\le\dfrac{3}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_2(p_3)=\begin{cases}\dfrac{P-\frac{2}{60}}{\frac{3}{60}-\frac{2}{60}};\ \dfrac{2}{60}\le p_3\le\dfrac{3}{60}\\ 1;\ \dfrac{3}{60}\le p_3\le\dfrac{5}{60}\\ \dfrac{\frac{6}{60}-P}{\frac{6}{60}-\frac{5}{60}};\ \dfrac{5}{60}\le p_3\le\dfrac{6}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_3(p_3)=\begin{cases}\dfrac{P-\frac{3}{60}}{\frac{5}{60}-\frac{3}{60}};\ \dfrac{3}{60}\le p_3\le\dfrac{5}{60}\\ 1;p_3=\dfrac{5}{60}\\ \dfrac{\frac{6}{60}-P}{\frac{6}{60}-\frac{5}{60}};\ \dfrac{5}{60}\le p_3\le\dfrac{6}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_4(p_3)=\begin{cases}\dfrac{P-\frac{4}{60}}{\frac{5}{60}-\frac{4}{60}};\ \dfrac{4}{60}\le p_3\le\dfrac{5}{60}\\ 1;\ \dfrac{5}{60}\le p_3\le\dfrac{7}{60}\\ \dfrac{\frac{8}{60}-P}{\frac{8}{60}-\frac{7}{60}};\ \dfrac{7}{60}\le p_3\le\dfrac{8}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_5(p_3)=\begin{cases}\dfrac{P-\frac{6}{60}}{\frac{7}{60}-\frac{6}{60}};\ \dfrac{6}{60}\le p_3\le\dfrac{7}{60}\\ 1;p_3=\dfrac{7}{60}\\ \dfrac{\frac{9}{60}-P}{\frac{9}{60}-\frac{7}{60}};\ \dfrac{7}{60}\le p_3\le\dfrac{9}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_6(p_3)=\begin{cases}\dfrac{P-\frac{6}{60}}{\frac{7}{60}-\frac{6}{60}};\ \dfrac{6}{60}\le p_3\le\dfrac{7}{60}\\ 1;\ \dfrac{7}{60}\le p_3\le\dfrac{9}{60}\\ \dfrac{\frac{10}{60}-P}{\frac{10}{60}-\frac{9}{60}};\ \dfrac{9}{60}\le p_3\le\dfrac{10}{60}\\ 0;\ otherwise\end{cases}$$

$$\mu_7(p_3)=\begin{cases}\dfrac{P-\frac{9}{60}}{\frac{10}{60}-\frac{9}{60}};\ \dfrac{9}{60}\le p_3\le\dfrac{10}{60}\\ 1;\ \dfrac{10}{60}\le p_3\le\dfrac{11}{60}\\ 0;\ otherwise\end{cases}$$

Thereby, both kinds of fuzzy numbers are obtaining from above LFMFs' of truth degrees set as follows:

$$T_rFN\ \widetilde{\alpha}\left(\mu_1(p_1)\right)=(a,\alpha,\beta)=\left(\frac{26}{60},\frac{25}{60},\frac{27}{60}\right)$$

$$T_pFN\ \widetilde{\alpha}\left(\mu_2(p_1)\right)=(a^L,a^U,\alpha,\beta)=\left(\frac{27}{60},\frac{29}{60},\frac{26}{60},\frac{30}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_3(p_1)\right)=\left(\frac{29}{60},\frac{27}{60},\frac{30}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_4(p_1)\right)=\left(\frac{29}{60},\frac{31}{60},\frac{28}{60},\frac{32}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_5(p_1)\right)=\left(\frac{31}{60},\frac{30}{60},\frac{33}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_6(p_1)\right)=\left(\frac{31}{60},\frac{33}{60},\frac{30}{60},\frac{34}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_7(p_1)\right)=\left(\frac{34}{60},\frac{33}{60},\frac{35}{60}\right)$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_1(p_2)\right)=(a,\alpha,\beta)=\left(\frac{8}{60},\frac{7}{60},\frac{9}{60}\right)$$

$$T_pFN\ \widetilde{\alpha}\left(\mu_2(p_2)\right)=(a^L,a^U,\alpha,\beta)=\left(\frac{9}{60},\frac{11}{60},\frac{8}{60},\frac{12}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_3(p_2)\right)=\left(\frac{11}{60},\frac{9}{60},\frac{12}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_4(p_2)\right)=\left(\frac{11}{60},\frac{13}{60},\frac{10}{60},\frac{14}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_5(p_2)\right)=\left(\frac{13}{60},\frac{12}{60},\frac{15}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_6(p_2)\right)=\left(\frac{13}{60},\frac{15}{60},\frac{12}{60},\frac{16}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_7(p_2)\right)=\left(\frac{16}{60},\frac{15}{60},\frac{17}{60}\right)$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_1(p_3)\right)=(a,\alpha,\beta)=\left(\frac{2}{60},\frac{1}{60},\frac{3}{60}\right)$$

$$T_pFN\ \widetilde{\alpha}\left(\mu_2(p_3)\right)=(a^L,a^U,\alpha,\beta)=\left(\frac{3}{60},\frac{5}{60},\frac{2}{60},\frac{6}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_3(p_3)\right)=\left(\frac{5}{60},\frac{3}{60},\frac{6}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_4(p_3)\right)=\left(\frac{5}{60},\frac{7}{60},\frac{4}{60},\frac{8}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_5(p_3)\right)=\left(\frac{7}{60},\frac{6}{60},\frac{9}{60}\right),T_pFN\ \widetilde{\alpha}\left(\mu_6(p_3)\right)=\left(\frac{7}{60},\frac{9}{60},\frac{6}{60},\frac{10}{60}\right),$$

$$T_rFN\ \widetilde{\alpha}\left(\mu_7(p_3)\right)=\left(\frac{10}{60},\frac{9}{60},\frac{11}{60}\right)$$

Now, LFRF formulation (4.2-5) and formulation (4.2-6) will be use to defuzzify fuzzily above fuzzy numbers of LFMFs' of truth degrees set to get crisp values as follows:

$$R(T_rFN)\left(\widetilde{\alpha}\left(\mu_1(p_1)\right)\right)=a+\frac{\beta-\alpha}{4}=\frac{26}{60}+\frac{\frac{27}{60}-\frac{25}{60}}{4}=\frac{26.5}{60}$$

$$R(T_pFN)\left(\widetilde{\alpha}\left(\mu_2(p_1)\right)\right)=\frac{a^L+a^U}{2}+\frac{\beta-\alpha}{4}$$

$$= \frac{\frac{27}{60}+\frac{29}{60}}{2} + \frac{\left(\frac{30}{60}-\frac{26}{60}\right)}{4} = \frac{29}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_3(p_1)\right)\right) = \frac{29}{60} + \frac{\left(\frac{30}{60}-\frac{27}{60}\right)}{4} = \frac{29.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_4(p_1)\right)\right) = \frac{\frac{29}{60}+\frac{31}{60}}{2} + \frac{\left(\frac{32}{60}-\frac{28}{60}\right)}{4} = \frac{31}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_5(p_1)\right)\right) = \frac{31}{60} + \frac{\left(\frac{33}{60}-\frac{30}{60}\right)}{4} = \frac{31.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_6(p_1)\right)\right) = \frac{\frac{31}{60}+\frac{33}{60}}{2} + \frac{\left(\frac{34}{60}-\frac{30}{60}\right)}{4} = \frac{33}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_7(p_1)\right)\right) = \frac{34}{60} + \frac{\left(\frac{35}{60}-\frac{33}{60}\right)}{4} = \frac{34.5}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_1(p_2)\right)\right) = a + \frac{\beta-\alpha}{4} = \frac{8}{60} + \frac{\left(\frac{9}{60}-\frac{7}{60}\right)}{4} = \frac{8.5}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_2(p_2)\right)\right) = \frac{a^L+a^U}{2} + \frac{\beta-\alpha}{4} = \frac{\frac{9}{60}+\frac{11}{60}}{2} + \frac{\left(\frac{12}{60}-\frac{8}{60}\right)}{4} = \frac{11}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_3(p_2)\right)\right) = \frac{11}{60} + \frac{\left(\frac{12}{60}-\frac{9}{60}\right)}{4} = \frac{11.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_4(p_2)\right)\right) = \frac{\frac{11}{60}+\frac{13}{60}}{2} + \frac{\left(\frac{14}{60}-\frac{10}{60}\right)}{4} = \frac{13}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_5(p_2)\right)\right) = \frac{13}{60} + \frac{\left(\frac{15}{60}-\frac{12}{60}\right)}{4} = \frac{13.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_6(p_2)\right)\right) = \frac{\frac{13}{60}+\frac{15}{60}}{2} + \frac{\left(\frac{16}{60}-\frac{12}{60}\right)}{4} = \frac{15}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_7(p_2)\right)\right) = \frac{16}{60} + \frac{\left(\frac{17}{60}-\frac{15}{60}\right)}{4} = \frac{16.5}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_1(p_3)\right)\right) = a + \frac{\beta-\alpha}{4} = \frac{2}{60} + \frac{\frac{3}{60}-\frac{1}{60}}{4} = \frac{2.5}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_2(p_3)\right)\right) = \frac{a^L+a^U}{2} + \frac{\beta-\alpha}{4} = \frac{\frac{3}{60}+\frac{5}{60}}{2} + \frac{\left(\frac{6}{60}-\frac{2}{60}\right)}{4} = \frac{5}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_3(p_3)\right)\right) = \frac{5}{60} + \frac{\left(\frac{6}{60}-\frac{3}{60}\right)}{4} = \frac{5.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_4(p_3)\right)\right) = \frac{\frac{5}{60}+\frac{7}{60}}{2} + \frac{\left(\frac{8}{60}-\frac{4}{60}\right)}{4} = \frac{7}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_5(p_3)\right)\right) = \frac{7}{60} + \frac{\left(\frac{9}{60}-\frac{6}{60}\right)}{4} = \frac{7.75}{60}$$

$$R(T_pFN)\left(\tilde{\alpha}\left(\mu_6(p_3)\right)\right) = \frac{\frac{7}{60}+\frac{9}{60}}{2} + \frac{\left(\frac{10}{60}-\frac{6}{60}\right)}{4} = \frac{9}{60}$$

$$R(T_rFN)\left(\tilde{\alpha}\left(\mu_7(p_3)\right)\right) = \frac{10}{60} + \frac{\left(\frac{11}{60}-\frac{9}{60}\right)}{4} = \frac{10.5}{60}$$

Finally, nine important/efficient points are obtained for each $p_{1,2,3}$ in each interval as follows:

$$\frac{25}{60} \le p_1 \le \frac{35}{60} \Rightarrow$$

$$p_1 \in \left\{\frac{25}{60}, \frac{26.5}{60}, \frac{29}{60}, \frac{29.75}{60}, \frac{31}{60}, \frac{31.75}{60}, \frac{33}{60}, \frac{34.5}{60}, \frac{35}{60}\right\}$$

$$\frac{7}{60} \le p_2 \le \frac{17}{60} \Rightarrow$$

$$p_2 \in \left\{\frac{7}{60}, \frac{8.5}{60}, \frac{11}{60}, \frac{11.75}{60}, \frac{13}{60}, \frac{13.75}{60}, \frac{15}{60}, \frac{16.5}{60}, \frac{17}{60}\right\}$$

$$\frac{1}{60} \le p_3 \le \frac{11}{60} \Rightarrow$$

$$p_3 \in \left\{\frac{1}{60}, \frac{2.5}{60}, \frac{5}{60}, \frac{5.75}{60}, \frac{7}{60}, \frac{7.75}{60}, \frac{9}{60}, \frac{10.5}{60}, \frac{11}{60}\right\}$$

5.2.5. Stage 5: Analyzing Cases

We have 9 cases for each p_i, then totally get $9^i, i \in \mathbb{N}$ cases to check. So, for current example $9^3 = 729$ cases are existed. Now, to check all cases to state condition $p = (p_{1h}, p_{2h}, p_{3h})^T \in \mathbb{R}^3, \sum_{i=1}^{N} p_{ih} = 1, p_{ih} \ge 0, i = 1,2,3, h = 1,2,\ldots,9$ of alpha-cut technique as follows:

For case (1,1,1): let $(p_{11}, p_{21}, p_{31}) = \left(\frac{25}{60}, \frac{7}{60}, \frac{1}{60}\right)$ then $\forall p_1, p_2, p_3 \ge 0$ and $p_1 + p_2 + p_3 = \frac{25+7+1}{60} = \frac{33}{60} \ne 1$, that is failed.

For case (7,8,8): let $(p_{17}, p_{28}, p_{38}) = \left(\frac{33}{60}, \frac{16.5}{60}, \frac{10.5}{60}\right)$ then $\forall p_1, p_2, p_3 \ge 0$ and $p_1 + p_2 + p_3 = \frac{33+16.5+10.5}{60} = \frac{60}{60} = 1$, that is pass and so on for entire other cases.

Thus, all of them are failed except three cases which (p_{17}, p_{28}, p_{38}), (p_{18}, p_{27}, p_{38}) and (p_{18}, p_{28}, p_{37}) are passes as follows:

$$(p_{17}, p_{28}, p_{38}) = \left(\frac{33}{60}, \frac{16.5}{60}, \frac{10.5}{60}\right)$$

$$(p_{18}, p_{27}, p_{38}) = \left(\frac{34.5}{60}, \frac{15}{60}, \frac{10.5}{60}\right)$$

$$(p_{18}, p_{28}, p_{37}) = \left(\frac{34.5}{60}, \frac{16.5}{60}, \frac{9}{60}\right)$$

5.2.6. Stage 6: Stochastic Transformation of Objective Coefficients via Expectation Weighted Summation Technique

We have three acceptance cases to apply stochastic transformation of objective coefficients on it to convert from STLPP to DTLPP by applying formulation (4.3-1) on Table (5.1-2) with entire three accepted cases of $p = (p_{17}, p_{28}, p_{38}) = \left(\frac{33}{60}, \frac{16.5}{60}, \frac{10.5}{60}\right) \in \mathbb{R}^3$, $p = (p_{18}, p_{27}, p_{38}) = \left(\frac{34.5}{60}, \frac{15}{60}, \frac{10.5}{60}\right) \in \mathbb{R}^3$, $p = (p_{18}, p_{28}, p_{37}) = \left(\frac{34.5}{60}, \frac{16.5}{60}, \frac{9}{60}\right) \in \mathbb{R}^3$. The EWS formulation (4.3-1) for each three above probability spaces $p = (p_1, p_2, p_3) \in \mathbb{R}^3$, then the obtained expected values of the problem after transformation of objective coefficients will be as follows:

$$Exp\ c_{ij}(\omega) = \sum_{k=1}^{3} c_{ij}(\omega_k)p_k = c_{ij}; i = 1,2,3; j = 1,2,3,4$$

$$Exp\ c_{11}(\omega) = c_{11}(\omega_1)p_1 + c_{11}(\omega_2)p_2 + c_{11}(\omega_3)p_3 = c_{11}$$
$$= (7.98)\left(\frac{33}{60}\right) + (7.99)\left(\frac{16.5}{60}\right) + (8)\left(\frac{10.5}{60}\right) = 7.98625$$
$$Exp\ c_{12}(\omega) = c_{12}(\omega_1)p_1 + c_{12}(\omega_2)p_2 + c_{12}(\omega_3)p_3 = c_{12}$$
$$= (6)\left(\frac{33}{60}\right) + (5.99)\left(\frac{16.5}{60}\right) + (5.98)\left(\frac{10.5}{60}\right) = 5.99375$$
$$Exp\ c_{13}(\omega) = c_{13}(\omega_1)p_1 + c_{13}(\omega_2)p_2 + c_{13}(\omega_3)p_3 = c_{13}$$
$$= (9.99)\left(\frac{33}{60}\right) + (9.98)\left(\frac{16.5}{60}\right) + (10)\left(\frac{10.5}{60}\right) = 9.989$$
$$Exp\ c_{14}(\omega) = c_{14}(\omega_1)p_1 + c_{14}(\omega_2)p_2 + c_{14}(\omega_3)p_3 = c_{14}$$
$$= (8.88)\left(\frac{33}{60}\right) + (8.89)\left(\frac{16.5}{60}\right) + (9)\left(\frac{10.5}{60}\right) = 8.90375$$
$$Exp\ c_{21}(\omega) = c_{21}(\omega_1)p_1 + c_{21}(\omega_2)p_2 + c_{21}(\omega_3)p_3 = c_{21}$$
$$= (9)\left(\frac{33}{60}\right) + (8.98)\left(\frac{16.5}{60}\right) + (8.96)\left(\frac{10.5}{60}\right) = 8.9875$$
$$Exp\ c_{22}(\omega) = c_{22}(\omega_1)p_1 + c_{22}(\omega_2)p_2 + c_{22}(\omega_3)p_3 = c_{22}$$
$$= (11.98)\left(\frac{33}{60}\right) + (11.96)\left(\frac{16.5}{60}\right) + (12)\left(\frac{10.5}{60}\right) = 11.978$$
$$Exp\ c_{23}(\omega) = c_{23}(\omega_1)p_1 + c_{23}(\omega_2)p_2 + c_{23}(\omega_3)p_3 = c_{23}$$
$$= (13)\left(\frac{33}{60}\right) + (12.98)\left(\frac{16.5}{60}\right) + (12.96)\left(\frac{10.5}{60}\right) = 12.9875$$
$$Exp\ c_{24}(\omega) = c_{24}(\omega_1)p_1 + c_{24}(\omega_2)p_2 + c_{24}(\omega_3)p_3 = c_{24}$$
$$= (6.98)\left(\frac{33}{60}\right) + (6.96)\left(\frac{16.5}{60}\right) + (7)\left(\frac{10.5}{60}\right) = 6.978$$
$$Exp\ c_{31}(\omega) = c_{31}(\omega_1)p_1 + c_{31}(\omega_2)p_2 + c_{31}(\omega_3)p_3 = c_{31}$$
$$= (13.96)\left(\frac{33}{60}\right) + (13.98)\left(\frac{16.5}{60}\right) + (14)\left(\frac{10.5}{60}\right) = 13.9725$$
$$Exp\ c_{32}(\omega) = c_{32}(\omega_1)p_1 + c_{32}(\omega_2)p_2 + c_{32}(\omega_3)p_3 = c_{32}$$
$$= (9)\left(\frac{33}{60}\right) + (8.98)\left(\frac{16.5}{60}\right) + (8.96)\left(\frac{10.5}{60}\right) = 8.9875$$
$$Exp\ c_{33}(\omega) = c_{33}(\omega_1)p_1 + c_{33}(\omega_2)p_2 + c_{33}(\omega_3)p_3 = c_{33}$$
$$= (15.96)\left(\frac{33}{60}\right) + (15.98)\left(\frac{16.5}{60}\right) + (16)\left(\frac{10.5}{60}\right) = 15.9725$$
$$Exp\ c_{34}(\omega) = c_{34}(\omega_1)p_1 + c_{34}(\omega_2)p_2 + c_{34}(\omega_3)p_3 = c_{34}$$
$$= (4.99)\left(\frac{33}{60}\right) + (5)\left(\frac{16.5}{60}\right) + (4.98)\left(\frac{10.5}{60}\right) = 4.991$$

5.2.7. Stage 7: Form The Problem as DTLPP Standard Version

Thus, the DTLPP is obtained for $case$ (7,8,8) and it will be as following table:

Table 5.2-1. The Obtain DTLPP Data Distribution of An Electricity STLPPFI Application Problem for Case (7,8,8)

Power Plants	HR	DK	SI	HA	Supply Million Kw/h
	Cites				
G_1	7.98625	5.99375	9.989	8.90375	36
G_2	8.9875	11.978	12.9875	6.978	51
G_3	13.9725	8.9875	15.9725	4.991	42
Demand Million Kw/h	46	22	31	30	Total = 129 M Kw/h Balanced

5.2.8. Stage 8: Finding BFS for DTLPP via MMCM

The MMCM will be applied to find BFS for above obtained DTLPP via find minimum cost in matrix c_{ij}, then enter minimum availability or requirement values for it, reduce same value for others that is not enter, repeat this as long as end, to get BFS (Sharma, 1974; Reeb, James Edmund;Leavengood, Scott A, 2002; Winston, Wayne L;Goldberg, Jeffrey B, 2004) as follows:

Table 5.2-2. The DTLPP Solution Steps via MMCM to Getting BFS for Case (7,8,8)

Power Plants	HR	DK	SI	HA	Supply Million Kw/h
	Cites				
G_1	7.98625&14	5.99375&22	9.989	8.90375	36//14//0
G_2	8.9875&32	11.978	12.9875&19	6.978	51//19//0
G_3	13.9725	8.9875	15.9725&12	4.991&30	42//12//0
Demand Million Kw/h	46//32//0	22//0	31//12//0	30//0	Total = 129 M Kw/h Balanced

Therefore, BFS is obtained based on (Reeb, James Edmund;Leavengood, Scott A, 2002; Winston, Wayne L;Goldberg, Jeffrey B, 2004; Sharma, 1974; Sengamalaselvi, 2017) as follows:

Table 5.2-3. The BFS of An Electricity STLPPFI Application Problem for Case (7,8,8)

14	22	0	0
32	0	19	0
0	0	12	30

5.2.9. Stage 9: Finding Optimal Solution for DTLPP via MODI Method

The current solution is not optimal, so the total transporting cost is calculated as follows:

$$(7.98625 * 14) + (5.99375 * 22) + (8.9875 * 32)$$

$$+(12.9875 * 19) + (15.9725 * 12) + (4.991 * 30) = 1119.4325\ \boldsymbol{IQD}$$

Now, the MODI method will be applied, it is necessary to use to find optimal BFS of DTLPP. The optimal BFS could be found based on (Reeb, James Edmund;Leavengood, Scott A, 2002; Winston, Wayne L;Goldberg, Jeffrey B, 2004; Sengamalaselvi, 2017) as follows:

Table 5.2-4. The Optimal BFS of An Electricity STLPPFI Application Problem for Case (7,8,8)

0	10	26	0
46	0	5	0
0	12	0	30

Thus, the optimal solution is obtained, so the total transporting cost is calculated as follows:

$$(8.9875 * 46) + (5.99375 * 10) + (8.9875 * 12) +(9.989 * 26) + (12.9875 * 5) + (4.991 * 30) = 1055.594\ \boldsymbol{IQD}$$

5.2.10. Stage 10: Selecting Post Optimal Solution for DTLPP

By repeating steps from Stage 6 till getting optimal solution for $p = (p_{18}, p_{27}, p_{38}) = \left(\frac{34.5}{60}, \frac{15}{60}, \frac{10.5}{60}\right)$, then optimal solution is obtained, and to relying on previous information from the DM and his/her preferred priorities to solve the problem, where the total transporting cost is calculated as follows:

$$(8.988 * 46) + (5.994 * 10) + (8.988 * 12) +(9.9893 * 26) + (12.988 * 5) + (4.9908 * 30) = 1055.627\ \boldsymbol{IQD}.$$

Also, by repeating steps from stage 6 till getting optimal solution for $p = (p_{18}, p_{28}, p_{37}) = \left(\frac{34.5}{60}, \frac{16.5}{60}, \frac{9}{60}\right)$, then optimal solution is obtained, and to relying on previous information from the DM and his/her preferred priorities to solve the problem, where the total transporting cost is evaluated as follows:

$$(8.9885 * 46) + (5.9943 * 10) + (8.9885 * 12) +(9.9887 * 26) + (12.9885 * 5) + (4.9912 * 30) = 1055.663\ \boldsymbol{IQD}.$$

Finally, we have optimal solution among three cases which is $case\ (7,8,8)$, where the total transporting cost is $1055.594\ \boldsymbol{IQD}$, and analytic optimal solution for above problem via crisp values version is $1057\ \boldsymbol{IQD}$, which this solution shows efficiency of STLPPFI algorithm via closing numerical solution from analytic/exact solution.

Now, to selecting post optimal solution as final result among the obtained optimal solutions for a certain amount case will be as follows:

1. If importance rank of probability problem is p_2 then p_3 then p_1 from $p = (p_1, p_2, p_3) = \in \mathbb{R}^3$, then DM should select $case\ (7,8,8)$ as a post optimal solution for certain problem.
2. If importance rank of probability problem is p_1 then p_3 then p_2 from $p = (p_1, p_2, p_3) = \in \mathbb{R}^3$, then DM should select $case\ (8,7,8)$ as a post optimal solution for certain problem with little additional penalty cost which is $0.033\ \boldsymbol{IQD}$.
3. If importance rank of probability problem is p_2 then p_1 then p_3 from $p = (p_1, p_2, p_3) = \in \mathbb{R}^3$, then DM should select $case\ (8,8,7)$ as a post optimal solution for certain problem with little additional penalty cost which is $0.069\ \boldsymbol{IQD}$.

6. Post Optimality and Sensitivity Analysis

To check and select the post optimality solution as final result to DM among the obtained optimal solutions for a certain amount case, the DM should be considered importance rank of probability problem since each chose depends on importance of it where sometimes DM give additional penalty cost when his/her chose is a difficult certain case, as illustrated carefully in subsection (5.2-10). In addition, if the credibility degree of DM about information on probability distribution is had small accurate or short probability intervals (i.e., intervals of stage 2) with short estimate cost value interval (i.e., each row of table (3.1-2)) then we get more and more good solution result.

7. Explanation of The Ranking Function and Its Effectiveness in The Uncertainty

In this section, we will explain our choice of this type of the ranking function, and its effectiveness in finding a solution to the stochastic problem with fuzzy information on the probability distribution.

1. There are other types of ranking functions. Those are having weaknesses. We explain whither this type of the ranking function is more suitable for our work. For

example; Khan's ranking function (Khan, Izaz Ullah;Ahmad, Tahir;Maan, Normah, 2013) is good for directions, because when applied to the set of real number, this ranking function yields values which may lie out of the interval of the fuzzy number. It is not suitable to be used in our work. Mahdavi-Amiri and Nasseri's ranking function (Mahdavi-Amiri, N;Nasseri, SH, 2006) those types yield values which may lie out of the interval of fuzzy number. It is not suitable to be used in our work. The same of reason holds for Ebrahimnejad's ranking function (Ebrahimnejad, 2011). While the ranking function which we have used, transforms the fuzzy set into a value within the interval. This type is more suitable for our work, because its real value remains in the fuzzy number's interval.

2. In stochastic programming with fuzzy information in the probability distribution, first we defuzzifying the fuzziness on the probability distribution via ranking function. After that we solve the stochastic programming by using a stochastic approach. Anyway, random constraints ought to transform first to obtain the deterministic feasible set. Thus, among all those series transformation, we obtain a compromise solution to our stochastic problem.

8. Conclusions

The study considered STLPPFI model problems. Solving STLPPFI via new approach are proposed and discussed. The proposed approach of solving STLPPFI problems classified into several stages of solution process, and it contains two main transformations, which are fuzzy transformation on probability distribution and stochastic transformation of problem formulation. Truth Degrees technique on probability distribution was improved from linguistic hedges technique to be better than past. The method have been employed in the environment of LFMF, $TpFN$, $TrFN$, LFRF, Alpha-Cut technique with Truth Degrees technique on probability distribution, EWS technique and alpha-cut technique filterazation via second condition test of it. Those techniques were bases on process of determining optimal solution to STLPPFI, and it proved that emerge entire actual situations of real-life STLPPFI model problems. The study had been showed that solution of STLPPFI are more efficient and has so close solution to analytic/exact solution, specifically where the interval of each row of estimate costs are not long too, it showed that the theory was parallel with the algorithm of it . The algorithm of STLPPFI problems had been use to convert STLPPFI into its corresponding equivalent DTLPP via defuzzifying the probability distribution and derandomization randomness of problem formulation respectively. Although, MMCM and MODI respectively were simplified solving DTLPP to gotten optimal solution. The solution result of STLPPFI numerical example in electricity field was explained finding BFS, finding optimal solution for it and selecting post optimality solution for it manually via DM deciding. Moreover, the solution result of STLPPFI was evaluated and supports entire stated techniques and algorithms. Finally, the proposed approach takes solution of the STLPPFI problems at the lowest total transporting costs, least elapsed-time running and maximum transporting amount of ships.

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9. Appendix
9.1. Fuzzy Symbols

The table of meaningful of necessary mathematical symbols entire paper.

Mathematical Symbols	Means
$(\Omega, 2^{\Omega}, P)$	Probability Distribution Space
$\tilde{A}$	Fuzzy Set A
$\equiv$	Equivalent To
$\Leftrightarrow$	If and Only If
$\Rightarrow$	If-Then
$\cong$	Approximately Equal To
$=$	Equal To

$\neq$	Not Equal To
$\therefore$	Therefore
$\because$	Because
$\wedge$	And
$\vee$	Or
x	Unknown Variables
$\tilde{x}$	Fuzzy Unknown Variables
ω	Stochastic Variable
$\tilde{\omega}$	Fuzzy Stochastic Variable
$\vdots$	As Long As Vertically
$\cdots$	As Long As Horizontally
$\ddots$	As Long As Diagonally
$<$	Less Than To
$>$	Greater Than to
$\leq$	Less Than or Equal To
$\geq$	Greater Than or Equal To
$\preccurlyeq$	Almost Less Than or Equal To
$\succcurlyeq$	At least Greater Than or Equal To
$\forall$	For All
$\exists$	There Exist
$\nexists$	There Does Not Exist
$F(\mathbb{R})(\omega)$	The Set of All Trapezoidal Fuzzy Stochastic Discrete Number Events
$F(\mathbb{R})$	The Set of All Trapezoidal Fuzzy Numbers
$F(R)$	The Set of All Fuzzy Numbers
inf	Infimum
sup	Supremum
$\mathbb{N}$	Natural Numbers
$\mathbb{Z}$	Integer Numbers
P	Probability Distribution
p	Probability Element Value
$\mathbb{R}$	Real Numbers
$\mathbb{R}^n$	n-Dimensional Vector of Real Numbers
$R(F)$	Linear Ranking Function
S	The Set of All Feasible Solution
A^T	Transpose of Vector/Matrix A
X	Universal Set X
Ω	Universal Stochastic Set
$\in$	Belong To
π	Default/Stochastic/Known/Determinate Polyhedral Set
$\tilde{\pi}$	Fuzzy Polyhedral Set
$Min\ z$	Minimize Objective Function z
$Max\ z$	Maximize Objective Function z
Exp	Expectation Process
$Exp_{P\in\pi}$	Expectation of All p That Belong to Stochastic Polyhedral Set π
$Exp_{P\in\tilde{\pi}}$	Expectation of All p That Belong to Fuzzy Polyhedral Set $\tilde{\pi}$
T_pFN	Trapezoidal Fuzzy Numbers
T_rFN	Triangular Fuzzy Numbers
$(x, \mu_{\tilde{A}})$	Ordered Pear of Fuzzy Set
$\mu_{\tilde{A}}$	The Membership Function $\mu_{\tilde{A}}$ of Fuzzy Set $\tilde{A}$

9.2. The Algorithm Program Outline of STLPPFI

The STLPPFI algorithm explaining in detail and it will be applied to solve illustrate example in section 5. The algorithm program outline of STLPPFI will be as follows:

Input: input fuzzy information on probability distribution via three vectors b_i, d_i and α_i as credibility degree of DM about information on probability distribution space, vagueness level vector and alpha-cut level vector respectively, then input an acceptance vector p from acceptable matrix in each row select one value, then input estimates cost values matrix, then input availability constraints vector and input requirement constraints vector, and should be total availability satisfy total requirement as applying balance condition on it.

Step 1: Form the problem as formulation (3.1-1) where information of it is as formulation (2.2-1).

Step 2: To transform formulation (3.1-1) with formulation (2.2-1) into formulation (3.1-1) with formulation (2.2-2) use truth degrees algorithm which is contain three algorithms respectively as follows:

Step 2a: Use formulation (4.1-3) i.e., (use alpha-cut algorithm) to obtain alpha-cut technique probability interval from three vectors b_i, d_i and α_i as credibility degree of the DM about information on probability distribution space, vagueness level vector and alpha-cut level vector respectively to cover and determine fuzzy uncertainty information on probability distribution.

Step 2b: Build the truth degrees set on probability distribution via uses dividing each obtain alpha-cut technique probability interval to ten continuous subintervals or eleven equal distance points in each row of degrees of truth of fuzzy logical value.

Step 2c: Build fuzzy truth degrees polyhedral set via using LFMFs formulation (4.1-1) and formulation (4.1-2) i.e., shown fuzzy truth degrees polyhedral set via both $TpFN$ and $TrFN$ in each row as fuzzy truth degrees regions $TrFN1, TpFN2, TrFN3, TpFN4,$ $TrFN5, TpFN6, TrFN7$, where fuzzy vector forms contain $TrFN$ as $(a, alpha, betta)$, and for $TpFN$ as $(a-lower, a-upper, alpha, betta)$. Then convert fuzzy truth degrees polyhedral set to deterministic values vector or nine importance/efficient points in each fuzzy truth degrees regions after applying LFRF formulation (4.2-5) and formulation (4.2-6) (i.e., use LFRF algorithm) to deffuzzifier all seven fuzzy numbers to obtain deterministic matrix values.

Step 3: Test second condition of alpha-cut technique probability interval formulation (4.1-3) (i.e., use acceptance P algorithm) which is from deterministic vector p in each row, we take $p1, p2, p3 \ldots pn$ values to obtain acceptance vectors to check that STLPP formulation (3.1-1) with stochastic polyhedral set formulation (2.2-2) are acceptable for entire cases to go to next step or not (where n cases are passes via test $9^\wedge n$ cases via $sum(pi) = 1, \forall pi \geq 0$). Then create matrix to find and select locations of acceptance cases via create acceptance location vector, then collect pi as acceptances' vectors of p.
Step 4: Use formulation (4.3-1) for deterministic acceptance vectors that passes step 3 (i.e., use EWS algorithm) to convert (STLPP formulation (3.1-1) with stochastic polyhedral set formulation (2.2-2)) into (DTLPP formulation (3.2-1)). This step starts via inputting vector p as an acceptance vector and inputting estimates distribution matrix as estimates cost values Table (3.1-2) with $(1, k)$ deterministic acceptance vector face, inputting final obtain deterministic cost value matrix in Table (3.2-1). Then applying EWS algorithm process on matrix acceptance vector p to convert matrix to deterministic cost value matrix in Table (3.2-1).
Step 5: Solve DTLLP formulation (3.2-1) n times with all acceptance vectors that passes Step 3 via Step 5a to get IFS, where formulation (3.2-1) contains from obtained deterministic cost matrix in Table (3.2-1) with both $(1, n)$ and $(m, 1)$ deterministic vectors via inputting availability constraints vector and requirement constraints vector in Table (3.2-1) where should be total availability satisfy total requirement.
Step 5a: Solve DTLPP via MMCM.
Step 6: Find optimal solution among n cases of acceptance vectors that passes Step 3 via MODI.
Step 7: Select post optimal solution via DM deciding recommendation for a certain case.
Output: Optimal Solution that contain best BFS with minimum total cost transporting electricity objective function, with selection post optimal solution value ■.