



Constructing mathematical Models, by Interpolation Methods, of people's interest to listening to Quran's voice or music

ID No. 3509

(PP 271 - 286)

<https://doi.org/10.21271/zjhs.24.5.19>**Nazeera Sedeek Barznji**

College of Administration and Economics
Department of Statistics and Informatics
/Salahaddin University-Erbil
nazeera.kareem@su.edu.krd

Rezan Sadeq Kareem

Institute of Training and Educational,
Development /Department of Mathematics
/Sulaimania
rezansadeq@gmail.com

Received: 22/02/2020**Accepted: 09/10/2020****Published: 10/12/2020**

Abstract

This paper aims to build mathematical models for the effectiveness and interesting of listening to the Quran's Voice or music on people. Quran is the religious text of Islam, which Muslims considered to be a revelation from God Lord of the worlds by the Prophet Muhammad (Peace be upon him). The Quran contains 30 chapters, 114 Surah, 6236 verses. The sound waves of voice of the Quran has a specific frequency and a specific wavelength, and these waves propagate vibrating fields that affect the brain cells and achieve a rebalancing of them, which gives them great immunity in fighting diseases, malfunction in the work the cells, and the effect of hearing the Qur'an on these cells, re-program them again. Music is the work of a person and it is an art of sound in time that expresses ideas and emotions. The experiments were conducted on a random sample that includes large numbers of people of both genders, as it included the opinions of people from (4) years to (48) years old at all academic levels from kindergarten to doctorate, and from all levels of non-academic society and various Muslim nationalities in Kurdistan. For this purpose, the Interpolation Method that is used is one of the important concepts in applied mathematics, especially in Numerical Analysis, it is the process of deriving a simple function from a set of discrete data points, Interpolation Method has several types. Two types are used here, the first type used is Newton Forward Interpolation Formula; when the domain points are equally spaced, and the second type is Lagrange Interpolation Formulas; when the domain points are not equally spaced.

Results of both types of analyses of interpolations show that people's interest to listening Quran's Voice is more effective than people's interest to listening to music because the Quran's Voice model is a quadratic model, but the music model is a linear model.

Keywords: Differences, Interpolation (Newton-Forward, Lagrange), Quran, and Music.

1. Introduction [1] [2] [8] [12] [16]

Numerical analysis is the area of mathematics and computer science that creates, analyzes and implements numerical method for solving numerically the problems of continuous mathematics. The numerical point of view goes back to the earliest mathematical writings. A tablet from the Yale Babylonian Collection (YBC 7289), gives a sexagesimal numerical approximation of the square root of 2, the length of the diagonal in a unit square. Numerical analysis naturally finds application in all fields of engineering and the physical sciences, but in the 21st century also the life sciences, social sciences, medicine, business and even the arts have adopted elements of scientific computations. Before the advent of modern computers, numerical methods often depended on hand interpolation formulas applied to data from large printed tables. Since the mid-20th century, computers calculate the required functions instead, but many of the same formulas nevertheless continue to be used as part of the software algorithms. Especially during the second half of the twentieth century and continuing up to the present day, digital computers have grown in power and availability. Interpolation is the process of deriving a simple function from a set of discrete data points so



that the function passes through all the given data points and can be used to estimate data points between the given ones. It is the process of finding such a polynomial is called interpolation and the most important approaches used is the Newton interpolation formula and Lagrange interpolation formula. These formulas involve finding a polynomial of order $n-1$ that passes through the n data points. In this article, we first show how the polynomial pattern can be identified and determined the degree by a set of points. The Quran describes itself as "the discernment" (al-furqān), "the mother book and its heavenly archetypal form for Muslims the source of divine revelation throughout human history and is eternally preserved by God. It is the covenant of God with humankind which Prophet Muhammad (peace be upon him) also professed to have experienced this heavenly Quran, the number of parts of the Holy Quran is thirty parts. Number of Holy Quran Surah 114. The number of verses of the Holy Quran 6236 verses. in addition to having been the recipient of revelation. Music is an art of sound in time that expresses ideas and emotions in significant forms through the elements of rhythm, melody, harmony, and color. the tones or sounds employed, occurring in single line (melody) or multiple lines (harmony), and sounded or to be sounded by one or more voices or instruments, or both.

2. The goal of the paper

- 1- Create two equations (Models) for people's interest to Quran's voice and Music using Newton Forward Interpolation formula
- 2- Create two equations (Models) for people's interest to Quran voice and Music using Lagrange Interpolation formula

3. Materials and Methods

The work in this paper is mainly based on Interpolation methods for two types of data first type when the domain points are equally spaced (-Equal intervals) by Newton forward interpolation formula and second type when the domain points are not equally spaced like (-Unequal intervals) by Lagrange interpolation formulas] described below

4. Theoretical Aspect

4.1 Finite Forward Difference Formula ^{[9][10][11][18]}

A **finite difference** is a mathematical expression of the form $f(x+b) - f(x+a)$. If a finite difference is divided by $b-a$, one gets a difference quotient. The approximation of derivatives by finite differences plays a central role in finite difference methods for the numerical solution of differential equations, especially boundary value problems.

Certain recurrence relations can be written as difference equations by replacing iteration notation with finite differences.

Today, the term "Finite Difference" is often taken as synonymous with finite difference approximations of derivatives, especially in the context of numerical methods. Finite Difference approximations are finite difference quotients in the terminology employed above. Finite differences were introduced by Brook Taylor in 1715 and have also been studied as abstract self-standing mathematical objects in works by George Boole (1860), L. M. Milne-Thomson (1933), and Károly Jordan (1939). Finite differences trace their origins back to one of Jost Bürgi's algorithms (c. 1592) and work by others including Isaac Newton. The formal calculus of finite differences can be viewed as an alternative to the calculus of infinitesimals.

A **Finite Difference** has three types are commonly considered: [*forward*, *backward*, and *central*] finite differences.

We take the Finite Forward Difference because it is necessary for Newton forward interpolation formula.

A **Forward Difference** is an expression of the form of difference identity giving an interpolated value between tabulated points (y_n) in terms of the first value



(y_0) and the powers of the Finite Forward Difference symbol is Lambda (Δ). For $a \in [0, 1]$,

the formula states

$$\Delta f(x) = f(x + h) - f(x)$$

i.e. for example $\Delta y_0 = y_1 - y_0 \quad \dots \quad \Delta y_{n-1} = y_n - y_{n-1}$

Table (1) illustrate finite forward difference

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

First forward differences:

$$\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1, \Delta y_2 = y_3 - y_2, \Delta y_3 = y_4 - y_3 \text{ and } \Delta y_4 = y_5 - y_4$$

..... (1)

Second forward differences:

$$\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0 = y_2 - y_1 - (y_1 - y_0) = y_2 - y_1 - y_1 + y_0 = y_2 - 2y_1 + y_0$$

..... (2)

Third forward differences:

$$\Delta^3 y_0 = \Delta^2(\Delta y_0) = \Delta^2(y_1 - y_0) = \Delta^2 y_1 - \Delta^2 y_0 = \Delta(\Delta y_1) - [\Delta y_2 - 2\Delta y_1 - (y_1 - y_0)] = \Delta(y_2 - y_1) - y_2 + 2y_1 + y_0 = \Delta y_2 - \Delta y_1 - y_2 + 2y_1 + y_0 = y_3 - y_2 - y_2 + y_1 - y_2 + 2y_1 + y_0 = y_3 - 3y_2 - 3y_1 + y_0$$

..... (3)

Fourth forward differences

$$\Delta^4 y_0 = \Delta^2(\Delta^2 y_0) = \Delta^2[\Delta^2(y_1 - y_0)] = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 \quad \dots \dots \dots (4)$$



$$\Delta^5 y_0 = \Delta^3(\Delta^2 y_0) = \Delta^3[\Delta^2(y_1 - y_0)] = y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 \dots\dots\dots (5)$$

$$\vdots \qquad \qquad \qquad \vdots$$

For the nth differences

$$\Delta^n y_0 = \Delta^{n-1} y_1 - \Delta^{n-1} y_0 = y_n - n y_{n-1} + \frac{n(n-1)}{2!} y_{n-2} - \frac{n(n-1)(n-2)}{3!} y_{n-3} + \frac{n(n-1)(n-2)(n-3)}{4!} y_{n-4} + \dots + (-1)^n y_0 \dots\dots\dots (6)$$

Notice that this polynomial $P_n(x)$ is also composed of $n+1$ distinct polynomial terms, but each of degree $i, i=0,1,2,\dots,n$. Term by term, each polynomial is of one degree higher than the previous one.

4.2 Interpolation [3] [4] [5] [18]

Interpolation is important concept in numerical analysis. Quite often functions may not be available explicitly but only the values of the function at a set of points, called nodes, tabular points or pivotal points. Then finding the value of the function at any non-tabular point, is called interpolation. So in the following sections we discuss Newton’s Interpolation formula which depend on difference operator and Lagrange’s Interpolation formula which they depend on Vandermonde's determinant. Interpolation of operators and some general approaches to the construction of numerical methods. The construction of numerical methods for solving mathematical problems written as $Ax = y$, where (x) and (y) are elements of certain sets $(X, Y$ and $A)$, may replace $(X, Y$ and $A)$, or only some of these three objects, by other objects that are convenient for calculating purposes. This replacement should be carried out in such a way that the solution to the new problem

$$\tilde{y} = A\tilde{x}$$

Consisting of finding \tilde{y} or \tilde{x} , is in some sense close to the solution of the original problem.

One of the methods for replacing (A) by an approximation (\tilde{A}) is the use of interpolation of

operators. The problem of the interpolation of operators has various formulations. A linear interpolation operator $L_1(F; x)$ for a given operator is written as

$$L_1(F; x) = F(x_0) + F(x_0, x_1)(x - x_0) \dots\dots\dots (7)$$

where x_0, x_1 are the interpolation nodes and $F(x_0, x_1)$ is the first-order divided-difference operator. The latter is defined as the linear operator for which

$$F(x_0, x_1)(x_0 - x_1) = F(x_0) - F(x_1) \dots\dots\dots (8)$$

The given definition of the divided-difference operator can be made concrete in a number of cases. Using linear interpolation (8), the "secant method" for the equation $F(x) = 0$ can be

$$\text{written as: } x_{n+1} = x_n - F^{-1}(x_{n-1}, x_n)F(x_n) \dots\dots\dots (9)$$

Where $F^{-1}(x_{n-1}, x_n)$ is the operator inverse to $F(x_{n-1}, x_n)$.



The formulation of the problem about interpolation functional, by the theory of approximation methods, is as follows. Let $\{\varphi_i(x)\}_{i=0}^n$ be some fixed functional, defined on (X) . A

functional $L_n[F; x]$ is called an interpolation functional polynomial for a given

functional $F(x)$ and system of points $\{x_k\}$ in X if the relations

$$L_n[F; x_k] = [F(x_k)], L_n[\varphi_i; x] = \varphi_i(x) \quad i = 0, 1, 2, \dots, n. \quad \dots\dots\dots (10)$$

Interpolation of functional is used in the construction of approximate methods for computing continual integrals, in finding extrema of functional, and in a number of other cases. e.g., approximate interpolation formulas for computing continual integrals have the form

$$\int_x F(x) d\mu(x) \approx \int_x L_n[F; x] d\mu(x) \quad \dots\dots\dots (11)$$

Where the integral over the interpolation polynomial $L_n[F; x]$ with respect to a certain

measure μ can be computed exactly or can be reduced to a finite-dimensional integral. When X is the space $C[a, b]$ of continuous functions on an interval $[a, b]$, then $L_1[F; x]$

can be represented by a Satieties integral,

$$L_1[F; x] = F(x_0) + \int_a^b \frac{x(T)-x_0(T)}{x_1(T)-x_0(T)} dTF[x_0(.) + x(. , T)(x_1(.) - x_1(.)) \dots\dots\dots (12)$$

where $x_0(t), x_1(t)$ are the interpolation nodes while

$$X(t, T) = \begin{cases} 1, & T > t \\ 0, & T \leq t \end{cases} \quad \dots\dots\dots (13)$$

If $F(x)$ is a constant or a linear functional, then.

$$L_1[F; x] \equiv F(x) \quad \dots\dots\dots (14)$$

The use of interpolation in finding extremely values of functional may be illustrated by two interpolation analogues of the gradient method for finding a local unconditional minimum of a functional $F(x)$ that is defined on some Hilbert space. The first analogue is obtained if, in

the gradient method, $grad F(x_n)$ is replaced by $F(x_{n-1}, x_n)$, i.e.

$$x_{n+1} = x_n - \epsilon_n F(x_{n+1}, x_n), \epsilon_n > 0, \quad n = 1, 2, \dots \quad \dots\dots\dots (15)$$

The second analogue uses the gradient of the interpolation polynomials. From approximations x_{n-2}, x_{n-1}, x_n to an extreme x^* of $F(x)$ one constructs the quadratic interpolation

polynomial

$$L_2[F; x] = F(x_n) + F(x_{n-1}, x_n)(x - x_n) + F(x_{n-2}, x_{n-1}, x_n)(x - x_{n-1})(x - x_n) \quad \dots\dots\dots (16)$$

Where $F(x_{n-2}, x_{n-1}, x_n)$ is the second-order divided difference of $F(x)$ with respect to x_{n-2}, x_{n-1}, x_n The new approximation x_{n+1} is

$$\text{determined by } x_{n+1} = x_n - \epsilon_n grad L_2 F; x_n, \epsilon_n > 0 \quad n = 2, 3, \dots\dots\dots (17)$$



The interpolation methods (16), (17) used two initial approximations respectively. The use of interpolation of operators and functional in the construction of computational algorithms for solving concrete problems is based on the use of interpolation formulas with a small error. Such a kind or formulas must be constructed for individual classes of functional and operators, taking specific features of these classes into account. Suppose that the function $f(x)$ is known at $(N + 1)$ points $(x_0, f_0), (x_1, f_1), \dots, (x_N, f_N)$ where the pivotal

points x_i spread out over the interval $[a, b]$ satisfy $a = x_0 < x_1 < \dots < x_N = b$

and $f_1 = f(x_1)$ then finding the value of the function at any non tabular point

$X_s(x_0 < x_1 < x_N)$ is called interpolation. Interpolation is done by approximating the

required function using simpler functions such as, polynomials. Polynomial approximations assume the data as exact at the tabular points $(N + 1)$ and generate an N^{th} degree

polynomial passing through these $(N + 1)$ points. However, if the given data has some errors

then these errors also will reflect in the corresponding approximated function.

4.2.1 Interpolation for domain points are equally spaced (Equal intervals in x_n)): Newton Forward Interpolation Formula ^{[3] [4] [5] [6] [7] [18] [19] [20]}

a Newton polynomial, is an interpolation polynomial for a given set of data points. Suppose that we have the $(n+1)$ points $((x_0, y_0), (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n))$ where all of the x_i are

different. For simplicity, we consider the case where these x_i are uniformly spaced with $(\Delta x = x_{n+1} - x_n)$ for each k These interpolating points determine a unique polynomial of

degree n (or possibly lower, if the points happen to lie on such a curve). Another way to express the equation of this polynomial is with the Newton Forward Interpolating Formula:

$$\Delta y_0 = y_1 - y_0 \rightarrow y_1 = y_0 + \Delta y_0 \rightarrow y_1 = (1 + \Delta)y_0 \dots\dots\dots (18)$$

$$\Delta y_1 = y_2 - y_1 \rightarrow y_2 = y_1 + \Delta y_1 \rightarrow y_2 = (1 + \Delta)y_1 \rightarrow y_2 = (1 + \Delta)(1 + \Delta)y_0 \rightarrow$$

$$y_2 = (1 + \Delta)^2 y_0 \dots\dots\dots (19)$$

$$\Delta y_2 = y_3 - y_2 \rightarrow y_3 = y_2 + \Delta y_2 \rightarrow y_3 = (1 + \Delta)y_2 \rightarrow y_3 = (1 + \Delta)(1 + \Delta)^2 y_0 \rightarrow$$

$$y_3 = (1 + \Delta)^3 y_0 \dots\dots\dots (20)$$

$$\Delta y_3 = y_4 - y_3 \rightarrow y_4 = y_3 + \Delta y_3 \rightarrow y_4 = (1 + \Delta)y_3 \rightarrow y_4 = (1 + \Delta)(1 + \Delta)^3 y_0 \rightarrow$$

$$y_4 = (1 + \Delta)^4 y_0 \dots\dots\dots (21)$$

$$\Delta y_4 = y_5 - y_4 \rightarrow y_5 = y_4 + \Delta y_4 \rightarrow y_5 = (1 + \Delta)y_4 \rightarrow y_5 = (1 + \Delta)(1 + \Delta)^4 y_0 \rightarrow$$

$$y_5 = (1 + \Delta)^5 y_0 \dots\dots\dots (22)$$

$$y_6 = (1 + \Delta)^6 y_0 \dots\dots\dots (23)$$



$$y_7 = (1 + \Delta)^7 y_0 \dots\dots\dots (24)$$

$$\cdot \quad \cdot$$

$$y_n = (1 + \Delta)^n y_0 \dots\dots\dots (25)$$

An alternate form of this equation is using binomial coefficients where the binomial coefficient $\binom{n}{i}$ represents a polynomial of degree i in n

The derivative of Newton's forward difference formula gives Markoff's formulas

$$y_n = (1 + \Delta)^n y_0 \dots\dots\dots (26)$$

$$y_n = \sum_{i=0}^n \binom{n}{i} i^{n-i} \Delta^i y_0 \dots\dots\dots (27)$$

$$y_n = y_0 + n\Delta^1 y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_0 + \dots + \frac{n!}{n!} \Delta^n y_0 \dots\dots\dots (28)$$

4.2 2 Interpolation for domain points are not equally spaced ((unequal intervals)) [13] [14] [15] [17] [20] [21]

Unequal intervals of the Lagrange's Interpolation formula. The former employs differences while the latter utilizes the function values. We begin this unit by studying first Lagrange's Interpolation formula

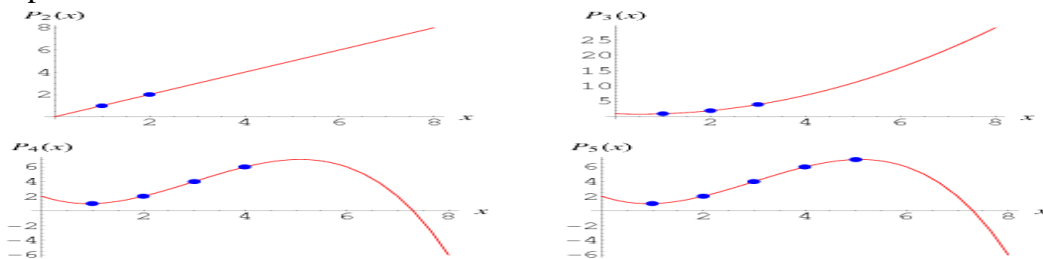


Figure (1) Samples of curve models in

Polynomial interpolation is the method of determining a polynomial that fits a set of given points. There are several approaches to polynomial interpolation, of which one of the most well-known is the Lagrangian method. The Lagrange interpolating polynomial is the polynomial $P(x)$ of

$$\text{degree} \leq (n - 1)$$

and is given by

$$P(x) = \sum_{i=1}^n P_j(x), \dots\dots\dots (29)$$

Where

$$P_j(x) = y_i \prod_{\substack{k=0 \\ k \neq j}}^n \frac{x-x_k}{x_j-x_k} \dots\dots\dots (30)$$

Written explicitly,

$$P(x) = \frac{(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} y_1 + \frac{(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} y_2 + \dots + \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} y_n \dots\dots\dots (31)$$

Lagrange interpolating are implemented in the Wolfram Language as Interpolating Polynomial [data, var]. They are used, for example, in the construction of Newton-Cotes formulas.



When constructing interpolating polynomials, there is a tradeoff between having a better fit and having a smooth well-behaved fitting function. The more data points that are used in the interpolation, the higher the degree of the resulting polynomial, and therefore the greater oscillation it will exhibit between the data points. Therefore, a high-degree interpolation may be a poor predictor of the function between points, although the accuracy at the data points will be "perfect."

For $n = 3$ points,

$$P(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \dots + \frac{(x-x_1)(x-x_2)}{(x_n-x_1)(x_n-x_2)}y_n \dots\dots\dots (32)$$

Note that the function $P(x)$ passes through the points (x_1, y_1) , as can be seen for the case $n = 3$

$$P(x_1) = \frac{(x_1-x_2)(x_1-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x_1-x_1)(x_1-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x_1-x_1)(x_1-x_2)}{(x_3-x_1)(x_n-x_2)}y_3 = y_1 \dots\dots\dots (33)$$

$$P(x_2) = \frac{(x_2-x_2)(x_1-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x_2-x_1)(x_2-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x_2-x_1)(x_2-x_2)}{(x_3-x_1)(x_3-x_2)}y_3 = y_2 \dots\dots\dots (34)$$

$$P(x_3) = \frac{(x_3-x_2)(x_1-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x_3-x_1)(x_3-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x_3-x_1)(x_3-x_2)}{(x_3-x_1)(x_3-x_2)}y_3 = y_3 \dots\dots\dots (35)$$

Generalizing to arbitrary n , $P(x_j) = \sum_{k=1}^n P_k(x_j) = \sum_{k=1}^n \delta_{jk} y_k = y_j \dots\dots\dots (36)$

The Lagrange interpolating polynomials can also be written using what Szegö (1975) called Lagrange's fundamental interpolating polynomials.

$$\prod_{k=1}^n (x - x_k) = \prod_{k=1}^n (x_j - x_k) = \left[\frac{d\pi}{dx} \right]_{\pi=x_j} = \prod_{k=1, k \neq j}^n (x_j - x_k) \dots\dots\dots (37)$$

$$\pi(x) \pi_v(x) = \frac{\pi(x)}{\pi'(x_v)(x-x_k)} \dots\dots\dots (38)$$

which satisfy

$$\pi_v(x_\mu) = \delta_{vp} \dots\dots\dots (39)$$

where δ_{vp} is the Kronecker delta. Now let $y_1 = P(x_1), \dots, y_n = P(x_n)$, then the expansion

$$P = \sum_{k=1}^n \pi_k(x) y_k = \sum_{k=1}^n \frac{\pi(x)}{(x-x_k)\pi'(x)} y_k \dots\dots\dots (40)$$

gives the unique Lagrange interpolating polynomial assuming the values y_k at x_k . More generally, let $(P_n(x))$ be the associated orthogonal polynomials on the interval $[a, b]$, and $(l_1(x), \dots, l_n(x))$, ..., the fundamental polynomials corresponding to the set of zeros of a polynomial $P_n(x)$. Then



$$\int_a^b L_v(x) L_\mu(x) d\alpha(x) = \beta_\mu \delta_{vp} \dots\dots\dots (41)$$

for $v, \mu = 1, 2, 3, \dots, n$, where (λ_v) are Christoffel numbers.

Lagrange interpolating polynomials give no error estimate. A more conceptually straightforward method for calculating them is Neville's algorithm.

Lagrange's classical formula of polynomial interpolation is given by the Lagrange Interpolating Formula. Suppose that we have the $n+1$ points $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$, where all of the x_i are different, though not necessarily uniformly spaced. These interpolating points then determine a unique polynomial of degree n (or possibly lower, if the points happen to lie on such a curve). One way to express the equation of this polynomial is with the Lagrange Interpolating Formula:

$$L_n = \sum_{i=0}^n y_i \prod_{k=0, k \neq i}^n \frac{(x-x_k)}{(x_i-x_k)} = \sum_{i=0}^n y_i \frac{(x-x_0)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} \dots\dots\dots (42)$$

Equivalently, if we write this formula without the summation notation, it becomes

$$L_n = \sum_{i=0}^n y_i \frac{(x-x_k)}{(x_i-x_k)} = y_0 \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} + y_1 \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} + \dots + y_n \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} \dots\dots\dots (43)$$

Notice that this polynomial is composed of $n+1$ distinct polynomial terms, each of degree n (provided $y_i \neq 0$). Either way, both of these are rather daunting expressions for students and consequently it is not surprising that many tend to miss some of the key underlying concepts.

5. Practical Aspect

5.1 Data Analysis

The data collected of this study from various Muslim nationalities in Kurdistan were conducted on a random sample that includes large numbers of people of both genders, as it included the opinions of people from (4) years to (48) years old at all academic levels from kindergarten to doctorate and from all levels of non-academic society.

X_i: represent ages , **Y₁** : Number of People's interest to listening Quran's Voice **Y₂** : Number of People's interest to listening Music **Y₃** : Number of People's interest to listening both Quran's Voice and Music

Table (2) Statistics for People's interest to listening Quran's Voice or Music and People's not interest to listening Quran's Voice and Music,

X_i age	Y₁ People's interest to listening Quran's Voice	Y₂ People's interest to listening Music	Y₃ People's interest to listening both Quran's Voice and Music	Total
4	16	5	9	30
8	64	9	17	90
12	144	13	43	200
16	256	17	67	340
20	400	21	79	500
24	576	25	99	700
28	784	29	87	900
32	1024	33	153	1210
36	1296	37	467	1800
40	1600	41	559	2200
44	1936	45	619	2600
48	2304	49	647	3000

In table (2) Statistics for people's interest to listening only Quran's voice, only music, both Quran's voice and music and total



5.2 Cronbach's Alpha

Table (3) Cronbach's Alpha for people's interest to listening only Quran's voice

Cronbach's Alpha Based on Standardized Items	N of Items
0.987	2

The Cronbach's alpha coefficient for the data about people's interest to listening only Quran's voice is (0.987), suggesting that the items have relatively high internal consistency.

Table (4) Cronbach's Alpha for people's interest to listening only music

Cronbach's Alpha Based on Standardized Items	N of Items
0.824	2

The Cronbach's alpha coefficient for the data about for people's interest to listening only music is (0.824) suggesting that the items have relatively high internal consistency.

5.3 Interpolations

5.3.1 Newton Forward Interpolation Formula is used when the domain points (x_i -Values) are equally spaced

Table (5) Newton Forward Interpolation Formula for people's interest to Quran's voice only

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$	$\Delta^6 y_i$	$\Delta^7 y_i$	$\Delta^8 y_i$	$\Delta^9 y_i$	$\Delta^{10} y_i$	$\Delta^{11} y_i$
X_{0-4}	$y_0 = 16$	48										
X_{1-8}	$y_1 = 64$		32									
X_{2-12}	$y_2 = 144$	80	0									
		112	0	0								
X_{3-16}	$y_3 = 256$	144	0	0								
		176	0	0	0							
X_{4-20}	$y_4 = 400$	208	0	0	0							
		240	0	0	0	0						
X_{5-24}	$y_5 = 576$	272	0	0	0	0						
		304	0	0	0	0	0					
X_{6-28}	$y_6 = 784$	336	0	0	0	0	0					
		368	0	0	0	0	0	0				
X_{7-32}	$y_7 = 1024$											
X_{8-36}	$y_8 = 1296$											
X_{9-40}	$y_9 = 1600$											
X_{10-44}	$y_{10} = 1936$											
X_{11-48}	$y_{11} = 2304$											

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_0 + \dots + \Delta^n y_0$$

$$y_n = 16 + n(48) + \frac{n(n-1)}{2!} (32) + \frac{n(n-1)(n-2)}{3!} (0) + \frac{n(n-1)(n-2)(n-3)}{4!} (0) + \dots + 0$$

$$y_n = 16 + n(48) + \frac{n(n-1)}{2!} (32) + (0) + (0) + \dots + 0$$

$$y_n = 16 + n(48) + \frac{n(n-1)}{2!} (32)$$



$$y_n = 16 + 48n + 16n(n - 1)$$

$$y_n = 16n^2 + 32n + 16 \dots\dots\dots (1)$$

$$x_1 = x_0 + 1h$$

$$h = 4, x_0 = 4$$

$$x_2 = x_0 + 2h$$

$$x_n = x_0 + nh$$

$$x_n = 4 + n4 \rightarrow n = \frac{x_n - 4}{4}$$

From (1) and (2)

$$y_n = 16 \left(\frac{x_n - 4}{4}\right)^2 + 32 \left(\frac{x_n - 4}{4}\right) + 16$$

$$y_n = (x_n - 4)^2 - 8(x_n - 4) + 16 \rightarrow y_n = x_n^2$$

From the result for table (5) of the analysis Newton Forward Interpolation formula for People interest to Quran’s Voice, we obtained a second degree equation (Quadratic Model)

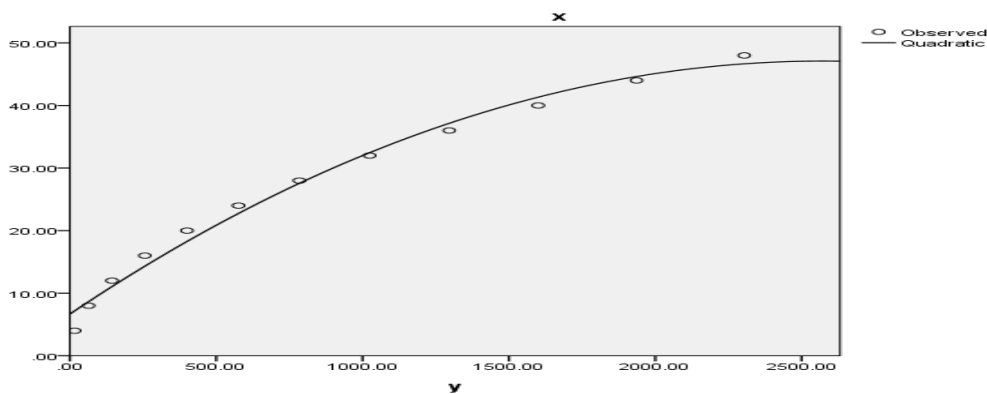


Figure (2) curve model of Quran's voice it emphasize for the model of Second degree (quadratic model)

Table (6) Newton Forward Interpolation Formula People’s interest to Music only

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$	$\Delta^6 y_i$	$\Delta^7 y_i$	$\Delta^8 y_i$	$\Delta^9 y_i$	$\Delta^{10} y_i$	$\Delta^{11} y_i$
$x_0=4$	$y_0=5$											
$x_1=8$	$y_1=9$	4	0									
$x_2=12$	$y_2=13$	4	0	0								
$x_3=16$	$y_3=17$	4	0	0	0							
$x_4=20$	$y_3=21$	4	0	0	0	0						
$x_5=24$	$y_5=25$	4	0	0	0	0	0					
$x_6=28$	$y_6=29$	4	0	0	0	0	0	0				
$x_7=32$	$y_7=33$	4	0	0	0	0	0	0	0			
$x_8=36$	$y_8=37$	4	0	0	0	0	0	0	0	0		
$x_9=40$	$y_9=41$	4	0	0	0	0	0	0	0	0	0	
$x_{10}=44$	$y_{10}=45$	4	0	0	0	0	0	0	0	0	0	0
$x_{11}=48$	$y_{11}=49$	4	0	0	0	0	0	0	0	0	0	0



$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_0 + \dots + \Delta^n y_0$$

$$y_n = 5 + n4 + 0 + 0 + 0 + 0$$

$$y_n = 5 + 4n \quad \dots\dots\dots (1)$$

$$x_1 = x_0 + 1h$$

$$h = 4, x_0 = 4$$

$$x_n = x_0 + nh$$

$$x_n = x_0 + nh$$

$$x_n = 4 + 4n \rightarrow n = \frac{x_n - 4}{4} \quad \dots\dots\dots (2)$$

From (1) and (2)

$$y_n = 5 + 4n$$

$$y_n = 5 + 4\left(\frac{x_n - 4}{4}\right)$$

$$y_n = x_n + 1$$

From the result of the Newton Forward Interpolation Formula for People's interest to Music we obtained a First degree equation (linear Model)

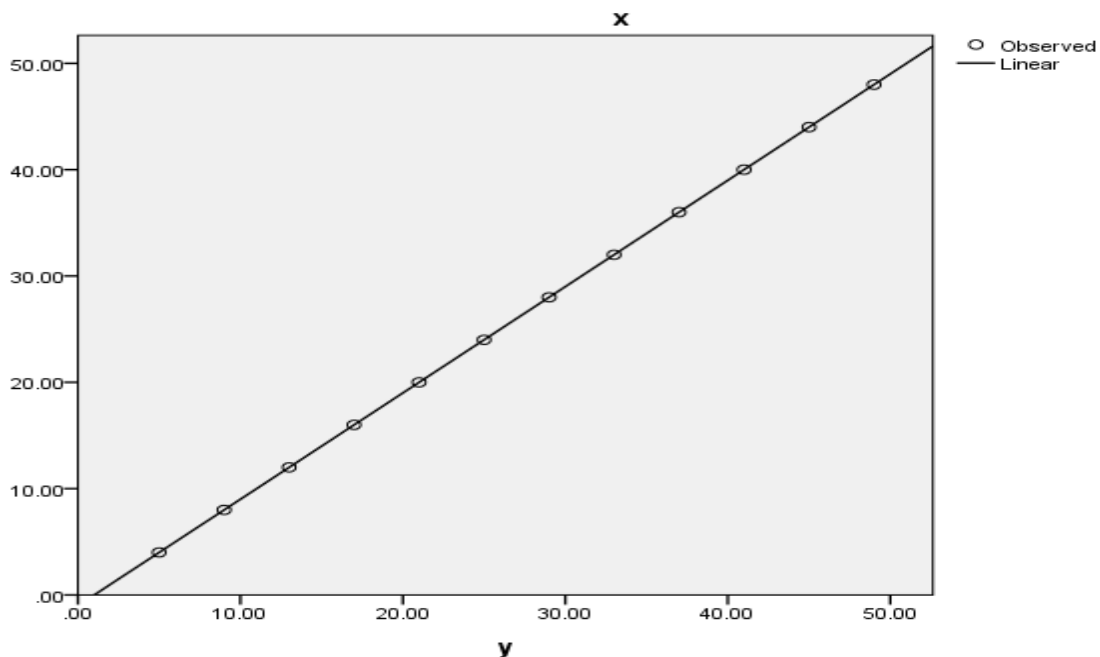


Figure (3) (striate line) model it emphasize for the model of first degree(linear model) by Newton Forward Interpolation of music

5.3.3 Lagrange Interpolation Formula: for the domain points $(x_i$ -Values) are not equally spaced

In the Lagrange Interpolation Formula there is a difference in the use of laws between finding the first degree equation and Finding the second degree equation

5.3.3.1: Lagrange Interpolation People's interest to Quran Voice Since the result of the equation of Quran Voice by Newton Forward is quadratic (second degree) so for finding the second-degree equation it needs the table contains three points like $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) as follows:

Table (7) Lagrange Interpolation People's interest to Quran's Voice only

x_i	y_i
$x_0 = 4$	$y_0 = 16$



$x_1 = 20$	$y_1 = 400$
$x_2 = 48$	$y_2 = 2304$

$$y_n = \sum_{i=0}^n L_k(x_n)y_i$$

$$L_0(x_n) = \frac{(x_n - x_1)(x_n - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x_n - 20)(x_n - 48)}{(4 - 20)(4 - 48)}$$

$$L_1(x_n) = \frac{(x_n - x_0)(x_n - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x_n - 4)(x_n - 48)}{(20 - 4)(20 - 48)}$$

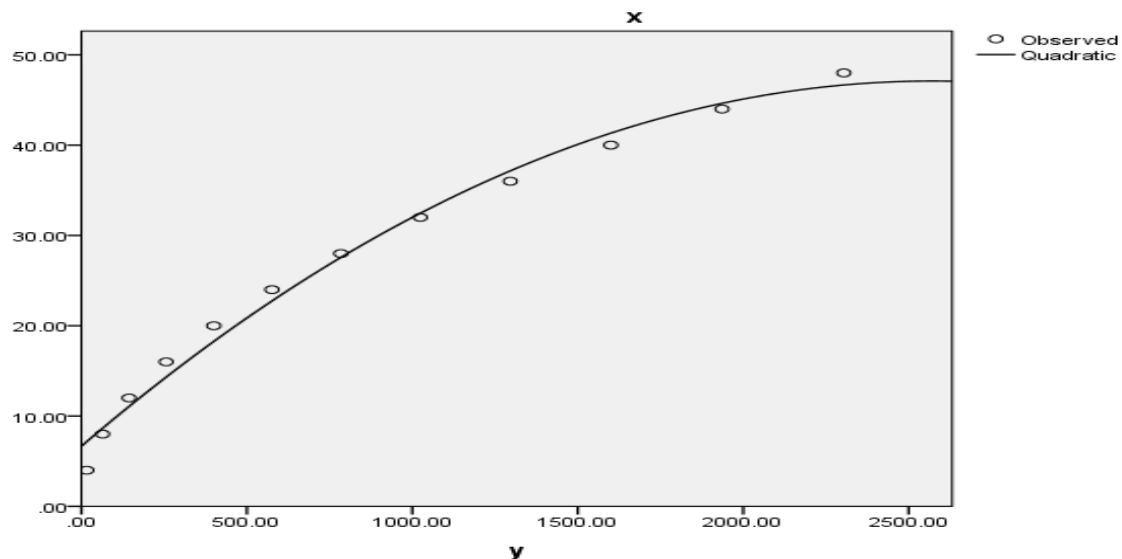
$$L_2(x_n) = \frac{(x_n - x_0)(x_n - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{(x_n - 4)(x_n - 20)}{(48 - 4)(48 - 20)}$$

$$y_n = \sum_{i=0}^n L_k(x_n)y_i$$

$$y_n = \frac{(x_n - 20)(x_n - 48)}{(4 - 20)(4 - 48)}(16) + \frac{(x_n - 4)(x_n - 48)}{(20 - 4)(20 - 48)}(400) + \frac{(x_n - 4)(x_n - 20)}{(48 - 4)(48 - 20)}(2304)$$

$$y_n = \frac{x_n^2 - 68x_n + 960}{704}(16) - \left(\frac{x_n^2 - 52x_n + 192}{348}\right)(400) + \frac{x_n^2 - 24x_n + 800}{1232}(2304)$$

$y_n = x_n^2$ the result of Quran Voice is also the equation of second degree (Quadratic)



Figure(5) of Quran's model it emphasize for the model of Send degree(quadratic model) by Lagrange Interpolation

5.3.3. 2: Lagrange Interpolation for people's interest to Music

In linear Lagrange Interpolation it needs the simple table only contains two points like (x_0, y_0) and (x_1, y_1) as follows:

Table(8)Lagrange Interpolation for people's interest to Music only

x_i	y_i
$x_0 = 4$	$y_0 = 5$
$x_1 = 48$	$y_1 = 49$

$$L_0(x_n) = \frac{(x_n - x_1)}{(x_0 - x_1)} = \frac{(x_n - 48)}{(4 - 48)}$$

$$L_1(x_n) = \frac{(x_n - x_0)}{(x_1 - x_0)} = \frac{(x_n - 4)}{(48 - 4)}$$

$$y_n = \sum_{i=0}^n L_k(x_n)y_i$$

$$y_n = \frac{(x_n - 48)}{(4 - 48)}(5) + \frac{(x_n - 4)}{(48 - 4)}(49)$$



$$y_n = \frac{(x_n - 48)}{-(44)} (5) + \frac{(x_n - 4)}{(44)} (49)$$

$$y_n = \frac{-5x_n + 240 + 49x_n - 196}{(44)}$$

$$y_n = \frac{44x_n + 44}{(44)}$$

$y_n = x_n + 1$ the result of Music is also the equation of first degree (Linear)

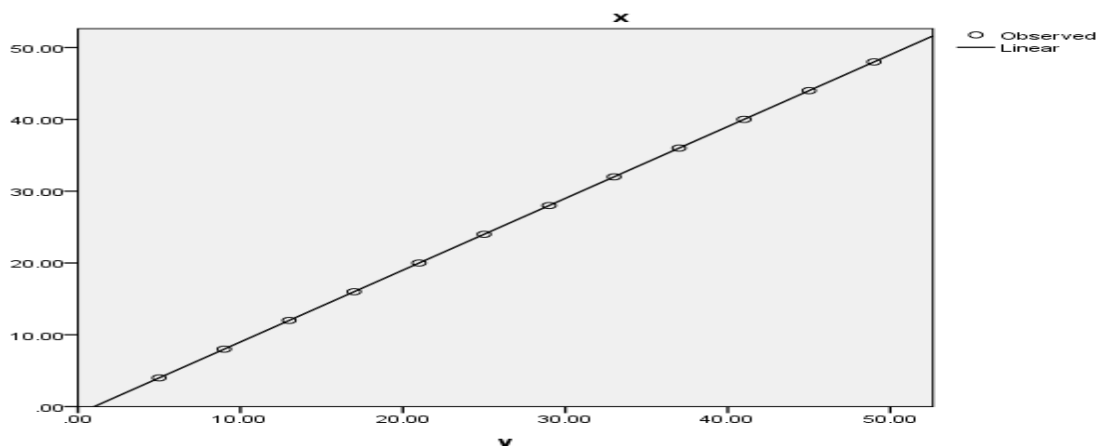


Figure (4) (striate line) model of music it emphasize for the model of first degree (linear model) by Lagrange Interpolation Formula

Conclusion

By the value of the result of the Cronbach's Alpha Based on standardized data analysis in both types of data: first for People's interest to listening to Quran's voice is (0.987), second for People's interest to listening to Music is (0.824) have relatively high internal consistency high reliability this evidence of the relevance of the data for the analysis (Note that a reliability coefficient of .70 or higher is considered "acceptable" in most social science research situations).

For small degrees of the interpolating polynomial (n < 20) the suitable interpolation formula Newton Forward Interpolation Formula and Lagrange Interpolation Formula. Both interpolations are requiring a special arrangement of the interpolating points to avoid numerical instabilities.

The interpret for the results of our study in the way that Newton Forward interpolation is in practice the most efficient and simple way to do equation(Model) interpolation. The functions in Newton Forward interpolation formulas are first function is for People's interest to Quran voice is an equation of second degree (Quadratic Model) and Second function is for People's Response to Music is an equation of first degree (Linear Model). But the functions in Lagrange interpolating formula are first function is for People's interest to Quran voice is a is an equation of second degree degree (Quadratic Model) and Second function is for People's interest to Music is an equation of first degree (Linear Model). It is clear in both method Newton Interpolation Formula and Lagrange Interpolation Formula have the same Equation for People's interest to Quran voice is (quadratic Model) and in both method Newton interpolation formula and Lagrange interpolation formula have the same Equation for People's interest to music is (a linear Model) It indicates that people's interest to is Quran's voice more than people's interest to music. It means that the Quran's voice has a high degree of interest at different ages of those involved in the preparation of data for this paper.

7. Recommendations

According to the conclusion reported above we may recommend the followings:



- 1- The Researcher recommend to emphasize if listening to Holy Qur'an has more positive effect than the listening to music in Psychological treatment.
- 2- The Researcher recommend to emphasize if listening to Holy Qur'an has more positive effect than the listening to music in medical treatment

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دروست كردنى ياساكانى بىركارى به رىگاناكانى (Interpolation) خوتيه لقورتان) بۇخواسى خه لك بۇ گوگرتن له دهنگى قورتان يان مۇسقىقا

ريزان صديق كريم

پهيمانگاي پاهينان و گه شه پيدانى بهروه رده ي -
بهشى بىركارى / سليمانى
rezansadeq@gmail.com

نظيرة صديق كريم برزنجى

كۆليژى به رپوه بردن و ئابوورى
بهشى ئاماروزاياريه كان / زانكۆ يلاح الدين
nazeera.kareem@su.edu.krd

پوخته

ئامانجى ئەم پەره يە پىكەينانى ھاوگىشە ي بىركارى به زياتر گىنگى دان به بىستنى دەنگى قورتان يان مۇسقىقا. قورتان برىتبه له نوسراو يىكى ئابنى كه به ئاشكرا له لايه بن يەزدانى مەزنه وه ئىردرا وه بۇ موسلمان له رىگانى پىغه مبه ر محمد (د.خ). وه قورتان پىكەتا وه له (۳۰) بهش و (۱۱۴) سورەت و (۶۲۳۶) ئايەت . دەنگى قورتان برىتبه له شه پۆله كانى دەنگى به له ره له رى ديارىكرا وه وه درىژى شه پۆلى ديارىكرا وه ئەو شه پۆلانه له كارىگه ريان له سه ر شانە كانى مېشك هه يه وه به رگه ربه كى گه وه ره بۇقه لاچۆى نه خو شى دەدات كه به هو ي تىكچوونى كارى شانە كانه وه تووش ئەبن وه كارىگه رى گوگرتن له قورتان له سه ر ئەو شانانە و ھاوسه نكى بۇده گه ر پىتته وه . به لأم مۇسقىقا كدرارىكى مرويه وه ونه رىكى دەنگه كه ده ر پىنى بىرو سوژه . بۇ ئەم پەره يە پارسى كرا له سامپلىكى هه ره مه كى ژماره يه كى زۆر له خه لك له هه ردوو په گه ز پىكەتا بوون كه له ته مه نى (۴) سالى تا (۴۸) سالى له هه موو ئاسته كانى ئەكادىمى له باخچە ي ساوايان تا دكتۆرا وه له هه موو ئاسته نا ئەكادىمىه كان و له هه موو خاوه ن رگه ز نامه يه كى موسلمان له كوردستان . بۇ ئەم مەبه سته رىگانى (Interpolation) به كار هىنرا كه به كىكه له بىروكه گرنگه كانى بىركارى پراكتىكى به تا بيه تى له شىكارى ژماره يى وه برىتبه له كدرارى دۆزىنه وه ي ھاوگىشە ي كۆمه لىك خالى جياواز وله چەند جورىكى هه يه وه ئىره دا تهنه دوو جورى به كار ئەه تىن جورى به كه ميان كاتىك جياوازي ئىوان خالە كان يە كسان بن رىگانى شىكردنه وه ي Newton Forward Interpolation Formula) (interpolation Formula) وه دووه ميان كاتىك جياوازي ئىوان خالە كان نا يە كسان بن (جياواز) بن رىگانى شىكردنه وه ي (Formula) Lagrange (Interpolation) ئەنجامى هه ردوو شىكردنه وه كه ي (Interpolation) ده رى خست كه گرنگى دان به گوگرتن له قورتان زياتره له گرنگى دان به گوگرتن له مۇسقىقا چونكه ھاوگىشە ي كارىگه رى گوگرتن له قورتان پله دوو ھاوگىشە ي كارىگه رى گوگرتن له مۇسقىقا پله يه كه .

كلىله ووشه: جياوازيه كان وانترپوله يشن (خۆتى هه لقورتان) (نيوتن فۆرورد ولاگرانج) (وقورتان ومۇسقىقا)

بناء نماذج رياضية ، باساليب التدخل (Interpolation) لاهتمام الناس للاستماع إلى صوت القرآن أو الموسيقى

ريزان صديق كريم

معهد التدريب و التطوير التربوي
القسم الرياضيات/السلمانية
rezansadeq@gmail.com

نظيرة صديق كريم برزنجى

كلية الادارة والاقتصاد. القسم الاحصاء والمعلوماتية
-جامعة صلاح الدين /اربييل
nazeera.kareem@su.edu.krd

ملخص

تهدف هذه الورقة إلى بناء نماذج لفعالية واستجابة الاستماع إلى صوت القرآن أو الموسيقى للناس. القرآن هو النص الديني للإسلام ، الذي يعتقد المسلمون أنه الوحي من الله رب العالمين للمسلمين عن طريق النبي محمد (ص) ، والقران تحتوي (30) جزءاً ، (114) سورة. و (6236) آية. إن صوت القرآن عبارة عن موجات صوتية لها تردد محدد وطول موجي محدد ، وتنتشر هذه الموجات تؤثر على خلايا الدماغ وتحقق إعادة توازنها ، مما يمنحها مناعة كبيرة في مكافحة الأمراض ، بسبب خلل في عمل الخلايا ، وتأثير سماع القرآن على هذه الخلايا ، وإعادة برمجةها مرة أخرى. اما الموسيقى هي عمل انساني وهي فن صوتي يعبر عن الأفكار والعواطف.لهذه الورقة وقد تم استفتاء من عينة عشوائية تضم أعداداً كبيرة من الناس من الجنسين ، تضمنت آراء الناس من العمر (4) سنوات إلى (48) سنة من جميع المستويات الأكاديمية من رياض الأطفال إلى الدكتوراه ومن جميع مستويات المجتمع غير الأكاديمي ومختلف الجنسيات المسلمة في كردستان ، ولهذا الغرض استخدم طريقة (Interpolation) هي واحدة من تلك المفاهيم المهمة في الرياضيات التطبيقية خاصة في التحليل العددي ، إنها عملية استخلاص المعادلة من مجموعة من نقاط البيانات المنفصلة. أن (Interpolation) يحتوي على عدة أنواع. ولكن يتم استخدام نوعين فقط ، يتم استخدام النوع الأول صيغة عندما تكون نقاط المجال متباعدة بشكل متساو (Newton Forward Interpolation Formula) والنوع الثاني صيغة (عندما لا تكون نقاط المجال متباعدة بشكل متساو) (Lagrange Interpolation Formula). أظهرت نتائج كلا النوعين من تحليلات (Interpolation) أن اهتمام الناس للاستماع إلى صوت القرآن أكثر فعالية من اهتمام الناس بالاستماع إلى موسيقى لأن معادله (نموذج) صوت القرآن هو معادله تريبيعه ولكن معادله الموسيقى هو معادله خطيه.

الكلمات الدالة: الفروقات والتدخل (نيوتن فور ورد ولاگرانج) والقران والموسيقى.