Using the Poisson queueing model to improve the queue in Periodic Vehicles Inspection PVI company in Erbil city

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Abstract

In this research we have dealt with the number of customers (cars) arrive at Periodic vehicles Inspection (PVI) company in Erbil city according to poisson process, service time follows an exponential distribution and the number of cars that require repair following a binomial distribution with a certain probability.

In this work the basic concepts of queueing theory are defined and the poisson queue model with multichannel homogeneous parallel servers in terms of steady state is derived.

In order to achive the objectives of this research, the data about the failure car components as (Tires, Lights, Brakes and Engine) which are inspected by PVI company are obtained From Erbil traffic directorate, from the available data. the probabilities of the failure car components (Tires, Lights, Brakes and engine), the probability of the number of cars in the system, the probability of the number of cars that require repair and they will return to the system are defined, also the measures of performance of the queuing system are determined.

Keywords: compound of Poisson process and binomial distribution, birth and death process, The Poisson queue model with multi servers.

1. Introduction

Queueing theory is the mathematical study of waiting lines which are a part of everyday life in different fields, because as a process it has several Important functions.

It is generally considered a branch of operation research because results are often used when making business decisions about the resources needed to provide the service, so queues are an essential way of dealing with flow customers when there are limited resources.

Queueing models are used so that queue length and waiting time can be predicted and make decisions on determining the line queue in order to avoid crowded in the general places.

Most queueing model deals with system performance in steady state and assume that the system has been operating with the same arrival rate, average service time and other characteristics. This research consists of three sections, the first section involves the basic concepts of queueing system, the second section specify application includes describing and analyzing of data and the third section shows the results determined by the practical part.

2. Methodology

This section is dedicated for the basic concepts of queueing model, the Poisson and Binomial distribution, the birth and death process in queue theory and the Poisson queue model $M|M|s$.

2.1 Queueing theory

The queueing theory means arrival customers at system service channels asking for a certain service, when the system is busy or the service not available the customers will wait in queue until they take their service. The queueing systems are included awaiting line and service channels, for convenience they represented with three characteristics which are arrival distribution, service time distribution and number of servers. (HILLIER & LIEBERMAN, 2015)

2.1.1 Characteristics (Kendall notation) of queueing models

The queueing system is represented the following six main characteristics: (Smith, 2018)

1. The arrival (or inter arrival time) distribution.
2. The departure (or service time) distribution.
3. Number of servers.
4. Queue discipline.
5. The maximum number of customers allowed in the System.
6. Size of the calling source.

2.1.2 Measures of performance of queueing systems

The most commonly used measures of performance in queueing situation are:

1. The expected number of customers in system (Ls).
2. The expected number of customers in queue (Lq).
3. The expected waiting time in system (Ws).
4. The expected waiting time in queue (Wq).
5. The expected number of busy servers (ρ).
6. The utilization factor for (s) servers $\left(\frac{\rho}{s}\right)$.

Where λ, μ are arrival and departure rates and $\rho = \frac{\lambda}{\mu}$.

The relationship between Ls and Ws (also Lq and Wq) is known as little's formula, and is given as:

$$Ls = \lambda Ws$$

$$Lq = \lambda Wq$$

$$Ws = Wq + \frac{1}{\mu} \tag{1}$$

$$Ls = Lq + \frac{\lambda}{\mu}$$

When the maximum number of customers allowed in the system has been restricted to N, thus the effective arrival rate λ_{eff} is used instead of λ.

$$\lambda_{eff} = \lambda(1 - P_N) \tag{2}$$

For large N, the λ_{eff} is equivalent to λ. (Shamblin & Stevens, 1974) (Chakravarthy, et al., 2015)

2.2 Poisson process and binomial distribution

Suppose that $N(t)$ be the number (n) of occurrences of an event during $(0,t]$ is a Poisson process with mean λt. Suppose also that each occurrence of that event has a constant probability (p) of being recorded independently of each other. So, the number (k) of occurrences $Y(t)$, which are recorded follows a binomial distribution for a specific number of occurrences of $N(t)$, then (Medhi, 2004) (Haldar & Mahadevan, 2000) (Beichelt & Paul Fatti, 2002)

$$Pr(Y(t)=k)=\sum_{n=k}^{\infty}p(Y(t)=k|N(t)=n).p(N(t)=n)$$
$$=\sum_{n=k}^{\infty}C_k^n p^k(1-p)^{n-k}e^{-\lambda t}(\lambda t)^n/n!$$
$$=\frac{p^k e^{-\lambda t}}{k!}\sum_{n=k}^{\infty}\frac{(\lambda t)^n(1-p)^{n-k}}{(n-k)!}$$
$$Pr(Y(t)=k)=\frac{(\lambda pt)^k e^{-\lambda pt}}{k!} \qquad ,k=0,1,2,\ldots \tag{3}$$

Which indicates that $Y(t)$ is also Poisson process with parameter λp

2.3 The birth and death process in queue theory

It can be shown that the simple birth and death process represented with (S) servers queueing system with Poisson arrivals with parameter (λ) and exponential service time with parameter (μ).

The probability that (k) events occur between t and $t+h$ given that (n) events occurred at time t can be defined for arrival and departure customers in queue theory as (U. Narayan Bhat, 2002)

$$Pr[number\ of\ arrivals\ between\ t\ and\ t+h\ is\ k,\ given\ that\ the\ number\ of\ customers\ in\ the\ system\ at\ time\ t\ is\ n]$$
$$p(k,h|n,t)=\begin{cases}1-\lambda_n h+o(h) & k=0\\ \lambda_n h+o(h) & k=1\\ o(h) & k>1\end{cases}$$

Pure birth

The probability of (n) customers in the system during t time

$$P_n(t)=\frac{(\lambda t)^n e^{-\lambda t}}{n!} \qquad ,n=0,1,2,\ldots \tag{4}$$

Thus, the interarrival distribution is

$f(t)=\lambda e^{-\lambda t} \quad ,t>0$

And

$$pr[number\ of\ departures\ between\ t\ and\ t+h\ is\ k,\ given\ that\ the\ number\ of\ customers\ in\ the\ system\ at\ time\ t\ is\ n]$$
$$q(k,h|n,t)=\begin{cases}1-\mu_n h+o(h) & k=0\\ \mu_n h+o(h) & k=1\\ o(h) & k>1\end{cases}$$

Where

$$\lim_{h\to 0}\frac{o(h)}{h}=0$$

Pure death

The probability of n customers served after t time is

$$p_n(t)=\frac{(\mu t)^{N-n}e^{-\mu t}}{(N-n)!} \qquad ,n=0,1,2,\ldots,N \tag{5}$$

Thus, the service time distribution is (Medhi, 2004)

$$g(t) = \mu e^{-\mu t} \quad ,t > 0$$

2.4 The Poisson queue model $M|M|S$

Let the arrivals occur in Poisson process with parameter (λ), and the services times be independent and identically distributed random variable with an exponential distribution with mean $\left(\frac{1}{\mu}\right)$. Let there be (s) servers in the system working in parallel independently of each other. Arriving customers queue up in a single line in the order of their arrival. A server who is free takes the customer at the head of the queue for service, it should be noted that in this system all servers offer service approximately at the same rate, thus the number of customers are served per unit time (μ_n) is defined as

$$\mu_n = \begin{cases} n\mu & if \quad 0 \le n < S \\ s\mu & if \quad S \le n \le N \end{cases}$$

In order to define the probability of exactly (n) customers in the system P_n, the birth and death process assumptions are applied and the following equations must be solved (Taha, 2007)

$$P_0(t+h) = P_0(t)(1-\lambda h) + P_1(t)\mu h + 0(h) \quad ,n = 0$$

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t) \tag{6}$$

$$P_n(t+h) = P_n(t)(1-(\lambda+n\mu)h) + \lambda h P_{n-1}(t) + (n+1)\mu h P_{n+1}(t) + 0(h)$$
$$0 < n < s$$

$$P_n'(t+h) = -(\lambda+n\mu)P_n(t) + \lambda P_{n-1}(t) + (n+1)\mu P_{n-1}(t) \tag{7}$$

$$P_n(t+h) = P_n(t)(1-(\lambda+s\mu)h) + \lambda h P_{n-1}(t) + s\mu h P_{n+1}(t) + 0(h)$$
$$s \le n < N$$

$$P_n'(t) = -(\lambda+s\mu)P_n(t) + \lambda P_{n-1}(t) + s\mu P_{n+1}(t) \tag{8}$$

$$P_N(t+h) = P_N(t)(1-s\mu h) + \lambda h P_{N-1}(t) + 0(h) \quad ,n = N$$

$$P_N'(t) = -s\mu P_N(t) + \lambda P_{N-1}(t) \tag{9}$$

Allowing $t \to \infty$ and taking the steady state of equations, such that

$$\lim_{t\to\infty} P_n'(t) = 0 \quad \& \quad \lim_{t\to\infty} P_n(t) = P_n$$

Then the equations (6, 7, 8 and 9) are become

$$\lambda P_0 = \mu P_1$$
$$(\lambda+n\mu)P_n = \lambda P_{n-1} + (n+1)\mu P_{n+1}$$
$$(\lambda+s\mu)P_n = \lambda P_{n-1} + s\mu P_{n+1}$$
$$\lambda P_{N-1} = s\mu P_n$$

Thus, the limiting distribution of the number of customers in the system (P_n) is obtained:

$$P_n = \begin{cases} \dfrac{\left(\frac{\lambda}{\mu}\right)^n}{n!} P_0 & 0 \le n < s \\ \dfrac{\left(\frac{\lambda}{\mu}\right)^n}{s!\, s^{n-s}} P_0 & s \le n \le N \end{cases} \tag{10}$$

Where

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \sum_{n=s}^{N} \frac{\left(\frac{\lambda}{\mu}\right)^n}{s!\, s^{n-s}} \right]^{-1} \quad (11)$$

The expected number of customers in the system is

$$L_s = \sum_{n=0}^{N} n p_n \quad (12)$$

and in the queue is

$$L_q = \sum_{n=s}^{N} (n-s) p_n \quad (13)$$

$$Waiting\ time = \frac{Number\ of\ customers\ in\ line}{Number\ of\ customers\ served\ per\ unit\ time} \quad (14)$$

For a single server (s=1) and N customers allowed in the system (above) model becomes $M|M|1$, then the probability of the number of customers in the system is given by

$$P_n = \frac{1-\rho}{1-\rho^{N+1}} \rho^n \qquad 0 \le n \le N \quad (15)$$

$$E(n) = L_s = \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})} \qquad ,where\ \ \rho = \frac{\lambda}{\mu} \quad (16)$$

3. Application Part

This section dedicated for application, which includes the computations of the probability (P) of the failure car components (Tires, Lights, Brake and Engine), the probability of the number of customers (Cars) arriving at periodic vehicles inspection (PVI) company, the probability of the number of cars that require repair and the measures of performance of queueing system are defined.

3.1 Data Description

The data about the failure cars components as (Tires, Lights, Brake and Engine), which are inspected during 11 years (2010-2020) by PVI company obtained from the traffic directorate of Erbil city.

The PVI company consists of three fields each one has 6-8 parallel channels (servers), They offer equal services independently of each other, with a single special channel is assigned for cars that require repair, whenever a component part fails the customer (car) will return again to the system after their failure parts are repaired, figure (3.1). The customers arrive at the system according to poisson process with mean 55 customer per hour (0.928 customer per minute), and the service time follows an exponential distribution with mean 12 minutes, The mean service rate of any server is 5 per hour, then the mean service rate of the system with 8 servers is 40 per hour (0.67 customer per minute).

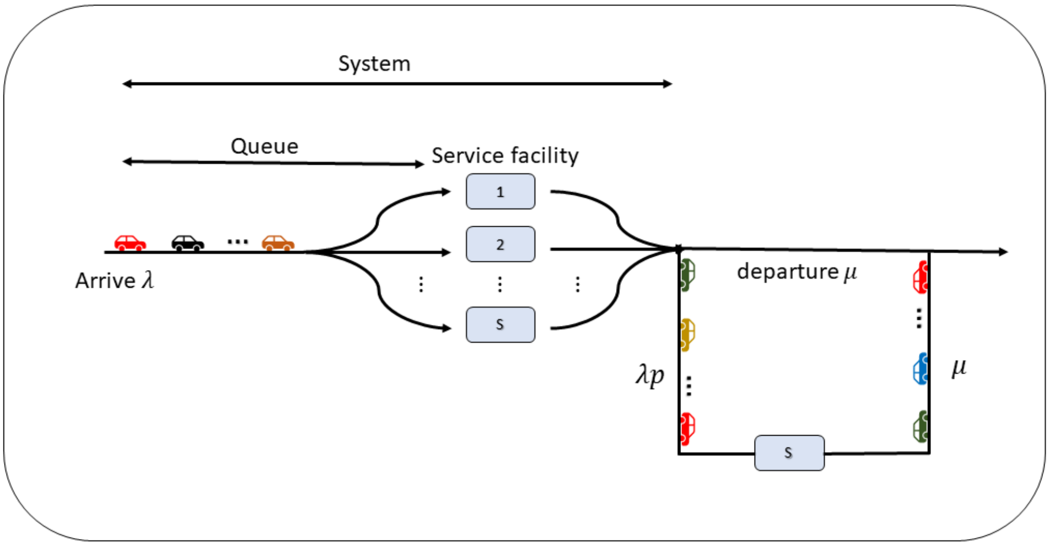

Figure 3.1 Showing the PVI System working

So, during 7 hours (8AM – 3PM, but at 2PM the entering customers to the system are no allowable) working daily approximately 300 cars are inspected at each field of the company and the failures are detected, (*table 3.1)*.

Table 3. 1 The probability of failure (Tire, Light, Brake, Engine) components

year	T_i	No. of Cars	Tire		Light		Brake		Engine	
			failure	$F(t)$	failure	$F(t)$	failure	$F(t)$	failure	$F(t)$
2010	1	171394	685	0.004	1199	0.007	1131	0.007	1268	0.007
2011	2	199632	1597	0.008	2795	0.014	2635	0.013	2954	0.015
2012	3	222057	2665	0.012	4663	0.021	4397	0.020	4930	0.022
2013	4	243576	3897	0.016	6820	0.028	6430	0.026	7210	0.030
2014	5	289045	5781	0.020	10117	0.035	9538	0.033	10695	0.037
2015	6	288412	6922	0.024	12113	0.042	11421	0.040	12805	0.044
2016	7	304960	8539	0.028	14943	0.049	14089	0.046	15797	0.052
2017	8	272490	8720	0.032	15259	0.056	14387	0.053	16131	0.059
2018	9	278413	10023	0.036	17540	0.063	16538	0.059	18542	0.067
2019	10	347003	13880	0.040	24290	0.070	22902	0.066	25675	0.074
2020	11	250828	11477	0.046	19433	0.077	18277	0.073	20700	0.083
Probability of Failure components			$P_{Tire} = 0.024$		$P_{Light} = 0.042$		$P_{Break} = 0.0396$		$P_{Engine} = 0.044$	

Where

$$P = \frac{\sum_{i=1}^{no.years} F(t_i)}{no.years}$$

3.2 Determined the probability of the number of customers (cars) in the system.

Table 3. 2 The probability of the number of cars being in the system

The number of cars in the system n	The probability of the number of cars in the system P_n
10	1.63E-07
20	3.94E-06
30	9.51E-05
40	2.30E-03
50	0.0555
55	0.2727

In the (*Table 3.2*) showed the number n and the probability (P_n) of the number of the customers (cars) and the probabilities of them are determined by equation (10), where the probability of zero customer in the system (p_0) is equal to $1.62E-11$, founded by equation (11).

3.3 Determinate the probability of the number of cars that require repair.

Table 3. 3 The probability of the number of cars that require repair their failure components

$P(Y_t = K)$	0	5	10	15	20	25	30
Tire	0.2671	0.0089	1.18E-06	1.31E-11	2.83E-17	1.78E-23	4.17E-30
Brake	0.1133	0.0463	7.50E-05	1.02E-08	2.69E-13	2.06E-18	5.92E-24
Light	0.0993	0.0544	1.18E-04	2.16E-08	7.64E-13	7.88E-18	3.03E-23
Engine	0.0889	0.0615	1.69E-04	3.89E-08	1.73E-12	2.26E-17	1.10E-22

Thus, the failure components (Tire, Light, Brake and Engine) are calculated by equation (3). *Table 3.3* shows the probabilities of the number of failure cars $(k = 0, 5, 10, 15, 20, 25, 30)$which are require to repair.

3.4 Define the measures of performance of a queue system.

Table 3. 4 The relationships between measures of performance of queueing system

Time (hour)	No. of customers in queue	Wq	Lq	Ls	Ws
1	15	0.38	20.63	31.63	0.58
2	30	0.75	41.25	52.25	0.95
3	45	1.13	61.88	72.88	1.33
4	60	1.50	82.50	93.50	1.70
5	75	1.88	103.13	114.13	2.08
6	90	2.25	123.75	134.75	2.45
7	50	1.25	68.75	79.75	1.45

The relationships between expected number of customers in system Ls, expected number of customers in queue Lq , expected waiting time in system Ws and expected waiting time in queue Wq are founded equation by equation (1), and the waiting time in queue of times $(t = 1, 2, \ldots, 7)$ hours are defined by equation (14). *Table 3.4* show the relationships between (Ls, Lq, Ws and Wq).

4. Conclusions

According to the results of application the following conclusions are found:

1. The number of failure components are increased annually, except in 2020, since the number of arrived customers to the system had lessen, because of covid-19.
2. The successive probabilities of the number of customers in the system are increased.
3. The successive probabilities of the number of cars that require repair their failures are decreased.
4. The number of customers in queue are increased with time, so the measures of performance of the queueing system are increased too.

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بەکارهێنانی مۆدێلی سڕەگرتنی پۆیسۆن بۆ باشترکردنی شێوازی سڕەگرتن لە کۆمپانیای پشکنینی ئۆتۆمبێل لە شاری هەولێر

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پوختە

لەم لێکۆڵینەوەیەماندا مامەڵەمان لەگەڵ ژمارەی کڕیاران (customers) (ئۆتۆمبێلەکان) کردووە کە دەگەنە کۆمپانیای پشکنینی ئۆتۆمبێلی (PVI) لە شاری هەولێر بەپێی پرۆسەی پۆیسۆن، دەرکەوتووە کە گەیشتنی کڕیارەکان (customer) بەدوای دابەشکردنی (Exponential)، وە ژمارەی ئەو ئۆتۆمبێلانەی کە پێویستیان بە چاککردنەوە هەیە بە ئەگەرێکی دنیایی.

لەم کارەماندا چەمکە بنەڕەتییەکانی تیۆری نۆرەگرتن و دۆخی نۆرەگرتنی پۆیسۆن لەگەڵ فرە کەناڵە هاو تەریبەکان دەناسێنێت کە لە ڕووی دۆخی جێگیرەوە بەدەستهێنراوە.

بەمەبەستی گەیشتن بە ئامانجەکانی توێژینەوەکەمان، داتای تایبەت بە لەکارکەوتنی پێکهاتەکانی ئۆتۆمبێل وەک (تایە و گڵۆپ و سوکان و بزوێنەر) کە لەلایەن کۆمپانیای PVI پشکنینیان بۆ دەکرێت لە بەڕێوەبەرایەتی هاتووچۆی هەولێرەوە وەردەگیرێت، لە داتا بەردەستەکان. ئەگەرەکانی لەکارکەوتنی پێکهاتەکانی ئۆتۆمبێل (تایە، گڵۆپ، سوکان و بزوێنەر)، ئەگەری ژمارەی ئۆتۆمبێلەکان لە سیستەمەکەدا، ئەگەری ژمارەی ئۆتۆمبێلەکان کە پێویستیان بە چاککردنەوە هەیە و دەگەڕێنەوە بۆ سیستەمەکە پێناسە کراوە، هەروەها. پێوەرەکانی ئەدای سیستەمی ڕیزکردن دیاری دەکرێن.

وشەی سەرەکی: - پێکهاتەی پرۆسەی پۆیسۆن و دابەشکردنی باینۆمیال ، پرۆسەی لەدایکبوون و مردن، مۆدێلی نزیکی پۆیسۆن بە فرەسێرڤەر.

استخدام نموذج صفوف انتظار بويسون لتحسين الطابور في شركة PVI لفحص المركبات في مدينة أربيل

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الملخص

في هذا البحث ، تعاملنا مع عدد العملاء (السيارات) الذين يصلون إلى شركة الفحص الدوري للمركبات (PVI) في مدينة أربيل وفقًا لعملية poisson، يتبع وقت الخدمة توزيعًا أسيًا وعدد السيارات التي تتطلب الإصلاح بعد التوزيع ذي الحدين مع احتمال معين.

في هذا العمل تم تعريف المفاهيم الأساسية لنظرية قائمة الانتظار واشتقاق نموذج قائمة انتظار بواسون مع خوادم متوازية متعددة القنوات ومتجانسة من حيث الحالة المستقرة.

من أجل تحقيق أهداف هذا البحث ، تم الحصول على البيانات الخاصة بمكونات السيارة المعطلة مثل (الإطارات والأضواء والمكابح والمحرك) التي تم فحصها من قبل شركة PVI من مديرية مرور أربيل ، من البيانات المتاحة. يتم تحديد احتمالات فشل مكونات السيارة (الإطارات والأضواء والمكابح والمحرك) واحتمالية عدد السيارات في النظام واحتمالية عدد السيارات التي تتطلب الإصلاح وستعود إلى النظام ، وكذلك يتم تحديد مقاييس أداء نظام الطابور.

الكلمة الرئيسية: - مركب عملية بواسون والتوزيع ذي الحدين ، عملية الولادة والوفاة ، نموذج قائمة انتظار بواسون بخوادم متعددة.