



Using the two type diseases Branching Process resulted by Chemical weapon

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Abstract

In this study we have dealt with the number of child's who are exposed the (eye, respiratory or both) diseases in the families whose at least one of their parents have the same diseases, which has caused by Chemical hit in Kurdistan of Iraq in 1988.

The diseases are become inherited and the offspring's have received the diseases from their affected parents and transmitted to successive generations. In order to a chive the objectives of this research, the data obtained from a sample of size (500) patient's family chosen randomly by stratified sampling, it described situations in which the families composed of members affected by the diseases.

From available data the transition probabilities and the expected length of time that the process spends in each state of the two types are determined; also the matrix of the expected number of offspring of the two types is defined.

Keywords: Finite state Markov Process, Two type branching process, Probability Generation Function.

1. Introduction

Branching processes have mainly been used to model the growth of population; they are widely used in biology and epidemiology to study the spread of infectious diseases and epidemics.

In this work the two type continuous-time homogenous branching process presented with a population of patients, since the individual carry two genes for unit character receives one from each parent, so they of either type will produce offspring's of possibly both types independently with the same direct descendant's probability distribution.

This research consists of three sections; the first section involves the basic concepts of Markov Process, two types branching process and the Probability Generation Function. The second section specify for application, it includes describing and analyzing of data, the third section shows the results determined by the practical part.

2: Methodology

This section illustrated some basic concepts of finite state Markov Process, transition probability matrix, two type branching process, probability generation function, mean of population size of the n-th generation for two types Branching Process.

2.1: Markov Processes

Of the different groups of stochastic processes, Markov processes occupy an important place. A stochastic process with a discrete parameter $\{X(t); t = 0, 1, 2, \dots\}$, or a stochastic process with a continuous parameter $\{X(t); t \geq 0\}$ is called a Markov process for any set of time periods $t_1 < t_2 < \dots < t_n$ if the $X(t_n)$ conditional distribution $\{X(t_1), X(t_2), \dots, X(t_n)\}$ of



information values depends on $X(t_{n-1})$ only and in the sense Accurate to any number of real numbers (X_1, X_2, \dots, X_n) . (Lawler, 2006) (Lefebvre, 2007)

$$P[X(t_n) \leq X_n | X(t_1) = X_1, X_2, \dots, X(t_{n-1}) = X_{n-1}] = P[X(t_n) \leq X_n | X(t_1) = X(t_{n-1}) = X_{n-1}] \quad \dots (1)$$

And that the equation means that if the state of the process is given at the present time, its state in the future does not depend on the movement of the process in the past.

It can be said that the Markov process has the following characteristic: -

Given the value of (X_t) , then the values of (X_E) where $(E > t)$ do not depend on the values of (X_u) where $(u < t)$, that is, the probability of any future state of the process when the current state is known is not related to the state in the past period.

The independent trials can be defined as the set of possible outcomes (E_1, E_2, \dots) finite or infinite and each of them is associated with a probability (P_k) . (Kirkwood, 2015)

$$P[(X_{j0}, X_{j1}, \dots, X_{jn})] = P_{j0}, P_{j1}, \dots, P_{jn} \quad \dots (2)$$

In Markov's theory, it takes into account simple generalizations, which lie in giving the opportunity for the outcome of any attempt to depend on the result of the immediate previous attempt, and that the result (E_k) is not related only with the constant probability (P_k) but with any pair (E_j, E_k) that matches the conditional probability (P_{jk}) .

Assuming that (E_j) occurs in some trials, the probability to (E_k) in the next attempt is (P_{jk}) and in addition to (P_{jk}) , we should give the probability (a_k) of the result (E_k) on the first attempt, Markov processes are categorized by: (Sericola, 2013)

- 1- The natural of the process, whether it is discrete or continuous parameter.
- 2- The natural of the scientific case space.

It is said that the real number (X) is a possible value or state of a random process $\{X(t), t \in T\}$ if there is a time (t) that belongs to (T) such that the probability $p\{X-h < X(t) < X+h\}$ is positive for each $(h > 0)$.

The set of possible values for a process in the space of that state is called The state space and the discrete space if it contains a finite or infinite number of states that can be computed. The not discrete state space is called the continuous space, and a Markov process with discrete state space is called a Markov chain. The set of integers $(0, 1, 2, \dots)$ is often used to mean the state space of a Markov chain. (Douc, 2018), (Bhat&Miller, 2002)

		State Space	
		Discrete	Continuous
Parameter Space	Discrete	Markov chain	Markov chain
	Continuous	Markov processes	Markov processes

The Markov process is described by the transition probability function and is often denoted by the $P(E, t/X_0, t_0)$ which represents the conditional probability that the state of the system in time (t) belongs to (E) if given that the system is in the state (X) in time $(t_0 < t)$.

It is said that Markov processes have stationary transitional probability or that they are homogeneous in time if they $P(E, t/X_0, t_0)$ depend on (t_0, t) only on difference $(t - t_0)$.



2.2: Two -State Markov Processes

In continuous time analog we may assume that the changes of state of mind are such that the resulting process is a Markov Process, To be specific, let $\{X(t); t \geq 0\}$ be a continuous parameter Markov Process with states $\{0, 1\}$.the transition probabilities defined as.(Kirkwood, 2015)

$$P_{ij}(t) = P[X(t) = j|X(0) = i] \quad i, j = 0,1$$

Hence for the two state Markov process $\{0, 1\}$, in which the process changes its state from 0 to 1 with rate (λ) and from 0 to 1 with rate (μ) , the transition probabilities are given by (Ross, 2019), (Medhi, 2014)

$$\left. \begin{aligned} P_{00}(t) &= \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \\ P_{01}(t) &= \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \\ P_{10}(t) &= \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t} \\ P_{11}(t) &= \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t} \end{aligned} \right\} \dots (3)$$

Farther, as $t \rightarrow \infty$

$$\left. \begin{aligned} \pi_0 &= \lim_{t \rightarrow \infty} P_{00}(t) = \lim_{t \rightarrow \infty} P_{10}(t) = \frac{\mu}{\lambda + \mu} \\ \pi_1 &= \lim_{t \rightarrow \infty} P_{01}(t) = \lim_{t \rightarrow \infty} P_{11}(t) = \frac{\lambda}{\lambda + \mu} \end{aligned} \right\} \dots (4)$$

Also the expected length of time $\mu_{ij}(t)$ in $(0, t)$ that the process spends in state (j) , having initially started from $X(0) = i (i = 0,1)$. Are given by

$$\left. \begin{aligned} \mu_{00}(t) &= \frac{\mu t}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2} [1 - e^{-(\lambda+\mu)t}] \\ \mu_{01}(t) &= \frac{\lambda t}{\lambda + \mu} - \frac{\lambda}{(\lambda + \mu)^2} [1 - e^{-(\lambda+\mu)t}] \\ \mu_{10}(t) &= \frac{\mu t}{\lambda + \mu} - \frac{\mu}{(\lambda + \mu)^2} [1 - e^{-(\lambda+\mu)t}] \\ \mu_{11}(t) &= \frac{\lambda t}{\lambda + \mu} + \frac{\mu}{(\lambda + \mu)^2} [1 - e^{-(\lambda+\mu)t}] \end{aligned} \right\} \dots (5)$$

The fraction of time in (t) that the Markov Process spends in states 0 or 1. These are given by

$$\left. \begin{aligned} \lim_{t \rightarrow \infty} \frac{\mu_{00}(t)}{t} &= \lim_{t \rightarrow \infty} \frac{\mu_{10}(t)}{t} = \frac{\mu}{\lambda + \mu} = \pi_0 \\ \lim_{t \rightarrow \infty} \frac{\mu_{01}(t)}{t} &= \lim_{t \rightarrow \infty} \frac{\mu_{11}(t)}{t} = \frac{\lambda}{\lambda + \mu} = \pi_1 \end{aligned} \right\} \dots (6)$$



And the distribution of the number of transitions a two-state Markov Process spent in each of the state (types) during every visit is negative exponential.

$$\left. \begin{aligned} f_0(t) &= \lambda e^{-\lambda t} & t > 0 \\ f_1(t) &= \mu e^{-\mu t} & t > 0 \end{aligned} \right\} \dots (7)$$

They related the process parameters λ and μ to the directly observable characteristics of the process the expected lengths of visits to each state $\{0,1\}$.

$$E(0) = \frac{1}{\lambda} \quad \text{and} \quad E(1) = \frac{1}{\mu} \quad \dots (8)$$

Thus crude estimates of the process parameters are obtaining

$$\lambda = \frac{1}{E(0)} \quad , \quad \mu = \frac{1}{E(1)} \quad \dots (9)$$

2.3: Two type branching processes

A main concept in spread biology is the variant increase proportion of different types of individuals. Two types branching processes supply a primary point for our discussion of this topic.

Take into consideration a branching Galton-Watson processes with a population of patients where tow different types may be distinguished, individuals of either type will produce offspring of possibly both types independently. (Kimmel, 2002)

Let U_n and V_n be a random variable representing the number of individuals of type (1) and type (2) ancestors respectively, in the n-th generation, so we can write

$$U_{n+1} = \sum_{j=1}^{U_n} X_j^1 + \sum_{j=1}^{V_n} X_j^2 \quad \dots (10)$$

$$V_{n+1} = \sum_{j=1}^{U_n} Y_j^1 + \sum_{j=1}^{V_n} Y_j^2 \quad \dots (11)$$

When the branching processes is one of the applications Markov sequent then the transition probability law of Markov process is:

$$P(X_j^i = K, Y_j^i = L) = P_i(k, l) \quad \dots (12)$$

$$k, l = 0, 1, 2, 3, \dots \quad \text{for} \quad j = 1, 2, 3, \dots \quad \text{and} \quad i = 1, 2$$

Where (X_j^i, Y_j^i) are independent identically distributed (iid) random vectors with probability mass functions $P_i(k, l)$ as above.

The simplest situation assume the process begins with a single individual then we define for incipient conditions

$$U_0 = 1 \quad \text{and} \quad V_0 = 0 \quad \dots (13)$$

Or

$$U_0 = 0 \quad \text{and} \quad V_0 = 1 \quad \dots (14)$$

And the probability extinction of individuals of the two types (Π^1, Π^2) are determined as follows:

For type (1)

$$\Pi^1 = P[U_n = 0, V_n = 0 | U_n = 1, V_n = 0] \quad \dots (15)$$



And for type (2)

$$\Pi^2 = P[U_n = 0, V_n = 0 | U_n = 0, V_n = 1] \quad \dots (16)$$

2.4: The offspring distribution

Let the process $X(t)$ be the number of direct descendants produced by and individual. The probability that an individual having the number (k) of affected offspring's has the poisson distribution with mean (λ) for type (1), (Gonzalez; et, 2010)

$$P[X(t) = k] = \frac{e^{-\lambda} \lambda^k}{k!} \quad k = 0,1,2,3, \dots \quad \dots (17)$$

And with mean (μ) for type (2),

$$P[X(t) = k] = \frac{e^{-\mu} \mu^k}{k!} \quad k = 0,1,2,3, \dots \quad \dots (18)$$

2.5: Probability Generation Function Relations for two types Branching Process.

In this section we show some relation for two-dimensional probability generation function of (U_n) and (V_n) offspring the population size of the n^{th} generation, as (Kirkwood, 2015)

$$\varphi_n^{(i)}(q_1, q_2) = E_n^i[q_1^k, q_1^l] = \sum_{k,l=0}^{\infty} P_i(k, l) q_1^k q_2^l \quad \dots (19)$$

$$0 \leq q_1, q_2 \leq 1 \quad \text{and} \quad i = 1,2$$

And their multiple step generalizations

$$\left. \begin{aligned} \varphi_n^{(1)}(q_1, q_2) &= \sum_{k,l \geq 0}^{\infty} p(U_n = k, V_n = l | U_0 = 1, V_0 = 0) q_1^k q_2^l \\ \varphi_n^{(2)}(q_1, q_2) &= \sum_{k,l \geq 0}^{\infty} p(U_n = k, V_n = l | U_0 = 0, V_0 = 1) q_1^k q_2^l \end{aligned} \right\} \quad \dots (20)$$

With $n \geq 0$, it is clear that the generation function of (4) is

$$\begin{aligned} \varphi_0^{(1)}(q_1, q_2) &= q_1 \\ \varphi_1^{(1)}(q_1, q_2) &= \varphi^{(1)}(q_1, q_2) \end{aligned}$$

and that of (5) is

$$\begin{aligned} \varphi_0^{(2)}(q_1, q_2) &= q_2 \\ \varphi_1^{(2)}(q_1, q_2) &= \varphi^{(2)}(q_1, q_2) \end{aligned}$$

And it can be start by generalizing the method used for the one - dimensional process that

$$\varphi_{n+m}^{(i)}(q_1, q_2) = \varphi_m^{(i)}(\varphi_n^{(1)}(q_1, q_2), \varphi_n^{(2)}(q_1, q_2)) \quad \dots (21)$$

2.6: Mean of Population size of the n-th generation for two types Branching Process.

The method of generation function is the most important tool in the study of branching processes; It is useful to determine the mean matrix of the n-th generation.



When (U_n) and (V_n) the number of offspring for the population composition of the n-th generation and consider the expectations, (Kimmel, 2002)

$$\left. \begin{aligned} \frac{\partial \varphi_n^{(1)}(q_1, q_2)}{\partial q_1} \Big|_{q_1=q_2=1} &= E[U_n | U_0 = 1, V_0 = 0] = m_{11}^{(n)} \\ \frac{\partial \varphi_n^{(2)}(q_1, q_2)}{\partial q_2} \Big|_{q_1=q_2=1} &= E[U_n | U_0 = 1, V_0 = 0] = m_{12}^{(n)} \\ \frac{\partial \varphi_n^{(1)}(q_1, q_2)}{\partial q_1} \Big|_{q_1=q_2=1} &= E[V_n | U_0 = 0, V_0 = 1] = m_{21}^{(n)} \\ \frac{\partial \varphi_n^{(2)}(q_1, q_2)}{\partial q_2} \Big|_{q_1=q_2=1} &= E[V_n | U_0 = 0, V_0 = 1] = m_{22}^{(n)} \end{aligned} \right\} \dots (22)$$

Then the expected matrix for n-th generation for two type branching process is,

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

So

$$M^{(n)} = \begin{bmatrix} m_{11}^{(n)} & m_{12}^{(n)} \\ m_{21}^{(n)} & m_{22}^{(n)} \end{bmatrix}$$

$$\begin{aligned} m_{ij} &= \frac{\varphi_{(1,1)}^1}{\partial q_j} & j &= 1, 2 \\ &= \lambda_{1j} & \dots & (23) \end{aligned}$$

And

$$\begin{aligned} m_{ij} &= \frac{\varphi_{(1,1)}^2}{\partial q_j} & j &= 1, 2 \\ &= \mu_{2j} & \dots & (24) \end{aligned}$$

Here:

$$M = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}$$

According to offspring's distribution of the two types (1) and (2) the probability generation function are given as, (Haccou, Jagers & Vatutin, 2005)

$$\begin{aligned} \varphi^{(1)}(q_1, q_2) &= \varphi(q_{11}) \cdot \varphi(q_{12}) \\ &= e^{\lambda_{11}(q_1-1) + \lambda_{12}(q_2-1)} \end{aligned}$$

$$\begin{aligned} \varphi^{(2)}(q_1, q_2) &= \varphi(q_{21}) \cdot \varphi(q_{22}) \\ &= e^{\mu_{21}(q_1-1) + \mu_{22}(q_2-1)} \end{aligned}$$

Also the expected number of offspring of the two types (1) and (2) successive generation size defined as below.



Then

$$G_n = (U_n \ V_n)$$

$$E(G_{n+s} | G_n) = G_n M^s \quad \dots (25)$$

3: Application

This section includes statistical study about the analysis of the data, we show here some of the techniques of Markov Processes and two Type Branching Processes for chemical attack diseases especially Eye and Respiratory diseases. The result of each method was performed by statistical package in Mat-lab.

3.1: Data Collection

In this study we have dealt with the number of child who are exposed to the (eye, respiratory or both) diseases in the families whose at least one of their parents are affected with the same diseases which has resulted by chemical weapon.

The diseases are become inherited and, so the offspring's have received the diseases from their affected parents and transmitted to successive generations, for this purpose the data have got from a sample of size (500) families chosen by stratified random sampling from the patients who are recorded by ministry of Martyr and Anfal of Kurdistan region.

Our attention on two diseases eye and respiratory, we noted some of the patients having more than one disease, the available data define in the following tables: -

Table (1): Represents the number of affected offspring according to their disease.

Disease	Eye	Respiratory	Total
Eye	222	162	384
Respiratory	230	237	467

From table (1) shown the offspring's number of diseases (Eye and Respiratory) where at least one of parents are affected.

Table (2): Represents the number of affected and unaffected parents by diseases (eye and respiratory).

Parents	Affected		Unaffected		Total
	Eye	Respiratory	Eye	Respiratory	
Father	246	320	27	23	616
Mather	237	324	36	19	616

3.2: The transition probability diseases matrix of Markov Processes.

The available data transformed to the transition probability disease matrix $p_{ij}(t)$ as defined below:-

$$S = \begin{bmatrix} 222 & 162 \\ 230 & 237 \end{bmatrix}$$

Where:

$$P_{ij}(t) = \frac{s_{ij}}{\sum_{j=1}^2 s_{ij}} \quad \forall \ i, j = 1, 2$$



Then

$$P_{ij}(t) = \begin{bmatrix} 0.5781 & 0.4219 \\ 0.4925 & 0.5075 \end{bmatrix}$$

When the distributions for type one (0) and type two (1) are exponential then their rates determined by equation (8) as

$$E(0) = \frac{222+230}{851} = 0.5311 \implies \lambda = 1.882$$

$$E(1) = \frac{162+237}{851} = 0.4689 \implies \mu = 2.1327$$

The distributions of the two types that the process spent in each state by equation (7) are

$$f(0) = (1.8829) e^{-(1.8829)t} \quad t > 0$$

$$f(1) = (2.1327) e^{-(2.1327)t} \quad t > 0$$

The transition probability $P_{ij}^{(t)}$ of Markov Processes $\{X(t)\}$ after (t) time by equation (3) as defined in table (3).

Table (3): The transition probability of $\{X(t)\}$ for (t) time

Time (t)	$P_{ij}^{(t)} = \begin{bmatrix} P_{00}^{(t)} & P_{01}^{(t)} \\ P_{10}^{(t)} & P_{11}^{(t)} \end{bmatrix}$
1	$P_{ij}^{(1)} = \begin{bmatrix} 0.5396 & 0.4605 \\ 0.5215 & 0.4785 \end{bmatrix}$
2	$P_{ij}^{(2)} = \begin{bmatrix} 0.5313 & 0.4687 \\ 0.5309 & 0.4691 \end{bmatrix}$
3	$P_{ij}^{(3)} = \begin{bmatrix} 0.5311 & 0.4689 \\ 0.5311 & 0.4689 \end{bmatrix}$
4	$P_{ij}^{(4)} = \begin{bmatrix} 0.5311 & 0.4689 \\ 0.5311 & 0.4689 \end{bmatrix}$
5	$P_{ij}^{(5)} = \begin{bmatrix} 0.5311 & 0.4689 \\ 0.5311 & 0.4689 \end{bmatrix}$
6	$P_{ij}^{(6)} = \begin{bmatrix} 0.5311 & 0.4689 \\ 0.5311 & 0.4689 \end{bmatrix}$

In table (3) showed the transition probability matrix in $(t = 4)$ is the steady state for each type , it means

$$\pi_0 = \lim_{t \rightarrow 4} P_{00}(4) = \lim_{t \rightarrow 4} P_{10}(4) = 0.5311$$

$$\pi_1 = \lim_{t \rightarrow 4} P_{01}(4) = \lim_{t \rightarrow 4} P_{11}(4) = 0.4689$$



3.3: The expected length of time that the Process spends in each state:

The expected length of time $\mu_{ij}(t)$ when the parameter of Markov Processes $\{X(t)\}$ spend by equation (5) it shown in table (4).

Time (t)	$\mu_{ij}^{(t)} = \begin{bmatrix} \mu_{00}^{(t)} & \mu_{01}^{(t)} \\ \mu_{10}^{(t)} & \mu_{11}^{(t)} \end{bmatrix}$
5	$\mu_{ij}^{(5)} = \begin{bmatrix} 2.7615 & 2.2385 \\ 2.5355 & 2.4645 \end{bmatrix}$
10	$\mu_{ij}^{(10)} = \begin{bmatrix} 5.4062 & 4.5938 \\ 5.2033 & 4.7967 \end{bmatrix}$
15	$\mu_{ij}^{(15)} = \begin{bmatrix} 8.0509 & 6.9491 \\ 7.8711 & 7.1289 \end{bmatrix}$
20	$\mu_{ij}^{(20)} = \begin{bmatrix} 10.6956 & 9.3044 \\ 10.5388 & 9.4612 \end{bmatrix}$
25	$\mu_{ij}^{(25)} = \begin{bmatrix} 13.3403 & 11.6597 \\ 13.2066 & 11.7934 \end{bmatrix}$
30	$\mu_{ij}^{(30)} = \begin{bmatrix} 15.9850 & 14.0150 \\ 15.8744 & 14.1256 \end{bmatrix}$
35	$\mu_{ij}^{(35)} = \begin{bmatrix} 18.6297 & 16.3703 \\ 18.5421 & 16.4579 \end{bmatrix}$
40	$\mu_{ij}^{(40)} = \begin{bmatrix} 21.2744 & 18.7256 \\ 21.2099 & 18.7901 \end{bmatrix}$
45	$\mu_{ij}^{(45)} = \begin{bmatrix} 23.9191 & 21.0809 \\ 23.8777 & 21.1223 \end{bmatrix}$
50	$\mu_{ij}^{(50)} = \begin{bmatrix} 26.5638 & 23.4362 \\ 26.5454 & 23.4546 \end{bmatrix}$
55	$\mu_{ij}^{(55)} = \begin{bmatrix} 29.2085 & 25.7915 \\ 29.2132 & 25.7868 \end{bmatrix}$

$$\lim_{t \rightarrow \infty} \frac{\mu_{00}(t)}{t} = \lim_{t \rightarrow \infty} \frac{\mu_{10}(t)}{t} = \frac{\mu}{\lambda + \mu} = \pi_0 = 0.5311$$

$$\lim_{t \rightarrow \infty} \frac{\mu_{01}(t)}{t} = \lim_{t \rightarrow \infty} \frac{\mu_{11}(t)}{t} = \frac{\lambda}{\lambda + \mu} = \pi_1 = 0.4689$$

3.4: The generation size of each type

The expected number offspring's affected by the two types eye and respiratory disease determined by equations (23), (24) and defined as bellow: -



$$M = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}$$

$$M = \begin{bmatrix} 0.4912 & 0.4060 \\ 0.5088 & 0.5940 \end{bmatrix}$$

Then the generation size of the number offspring by equation (25) it shown in table (5).

Expected of Generations	$E(G_{n+s} G_n) = G_n M^s$
1	$E(G_1) = (452 \quad 399)$
2	$E(G_2) = (425.0336 \quad 420.5180)$
3	$E(G_3) = (422.7361 \quad 422.3513)$
4	$E(G_4) = (422.5403 \quad 422.5075)$
5	$E(G_5) = (422.5236 \quad 422.5208)$
6	$E(G_6) = (422.5222 \quad 422.5219)$
7	$E(G_7) = (422.5220 \quad 422.5220)$
8	$E(G_8) = (422.5220 \quad 422.5220)$
9	$E(G_9) = (422.5220 \quad 422.5220)$
10	$E(G_{10}) = (422.5220 \quad 422.5220)$

4: Conclusions:

According to the results from the practical part we reach to the following conclusions: -

- 1- The transition probabilities of the two type diseases have stabilized at time (t=3), table (3).
- 2- The expected length of time that the process spends in each state of the two types are continually increasing with time until (t=55), table (4).
- 3- In the second generation the size of the first type has decreased, but the size of the second type has increased, after that the sizes of both types are no more changed, table (5).
- 4- The generation size stability of both types have attained at time (t=7), table (5).

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كلية الادارة و الاقتصاد - قسم الاحصاء / جامعة صلاح الدين-أربيل

ملخص

في هذه الدراسة تعاملنا مع عدد الأطفال الذين تعرضوا لأمراض (العيون أو الجهاز التنفسي أو كليهما) في العائلات التي تعاني أحد والديهم على الأقل من نفس الأمراض التي تسببها الضربة الكيميائية في كردستان العراق عام 1988. ويتلقى الأبناء هذه الأمراض بشكل وراثي من آبائهم المصابين وينتقل إلى الأجيال المتعاقبة. من أجل تحديد أهداف هذا البحث، تم الحصول على عينة من البيانات حجمها (500) مريض حيث تم اختيارها عشوائياً عن طريق أخذ العينة الطبقة حيث وصفت الحالات التي تتكون فيها العائلات من أفراد المصابين بالأمراض الواردة. من البيانات المتاحة تم تحديد احتمال الانتقال والمدة المتوقعة للوقت الذي تقضيه العملية في كل حالة من النوعين وايضا كما تم تحديد مصفوفة العدد المتوقع للنسل لكل النوع.

الكلمات المفتاحية: الحالة المنتهية لعملية ماركوف، العملية المتفرعة ذات نوعين، الدالة المولدة الاحتمالية.

به کارهیتانی پرۆسه ی لقرار بۆ دوو جوړ له نه خوښی دهرته نجامی چه کی کیمیایی

دره خشان جلال حسن

کۆلیژی کارگيري و ئابووری - به شی ئامار / زانکۆی سه لاهه ددين-هه ولتر

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کۆلیژی کارگيري و ئابووری - به شی ئامار / زانکۆی سه لاهه ددين-هه ولتر

پوخته

له م ليكۆلينه وه كار له سه ر ژماره يه ك مندال كراوه كه تووشی نه خوښی يه كانی (چاو، تهنگه نه نفسی يان هه ردوو نه خوښی) بووینه له م خيزانانه ی كه به لايه نی كه مه وه يه كيك له باوانيان هه مان نه خوښيان هه يه كه هوكاره كه ی ده گه رته وه بۆ ليدانی گازی کیمیایی له كوردستانی عيراق له سالی ١٩٨٨. كاتج ئه م نه خوښيانه به شیوه ی بوماوه یی ده گوازرتنه وه له باوانه وه بۆ منداله كان و نه وه ی يه ك به دوا ی يه ك. وه بۆ ده ست نيشان كردنی ئامانجه كانی ئه م تووژينه وه يه بژارده يه ك به قه باره ی (500) نه خوښ وه رگيراهه به شیوه ی هه په مه كی به به کارهیتانی بژارده ی هه په مه كی چینی كاتج ئه و خيزانانه ی تيدا وه سف كراوه كه ئه ندای تووش بووی به م نه خوښيانه ی تيدا يه. وه له م داتا يانه ی كه له به رده ست بوو تيايدا ئه گه رى گواسته وه وه ماوه ی چاوه رپوان كراوی ئه و كاته ی كه ئه م پرۆسه يه بۆ هه ريه كه له جوړه كانی نه خوښی به كاریده هیتیت ده ست نيشانكراوه وه هه روه ها ده ست نيشان كردنی ماتریكسی هه ژماری چاوه رپوان كراو بۆ نه وه ی هه ردوو جوړ.

ووشه ی سه ره کی: حالته ی دوا یی بۆ پرۆسه ی ماركوف، پرۆسه ی لقرار بۆ دوو جوړ، نه خسه ی ئه گه رى چيگير.