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Research Article

A Stochastic Analysis to Calculate the Mean Time to System Failure (MTSF) of A Two Identical Unit Parallel System

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Abstract

In this research two identical parallel electric generators with repair a failed one have been analyzed stochastically by two methods, first by method based on certain probability assumptions and second through three state Markov chain, classified as one recurrent (absorbing) state and two transient states. For the purpose of this study the data about the failure and repair time intervals of electric generators of the months november and december 2022 are obtained from the electricians of several places in Erbil city, whereas they supplied electricity to their lanes whenever the national electric power is interrupted. This research aimed to define the mean time to system failure by the two previous methods.

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1. Introduction

In the recent years, current industrial world has daily been facing with anew systems requiring high levels. In this regard, great attentions have been attracted by reliability and safety, which are very important aspects to many situations of our quality of life.

Systems are the collection of components or units arranged to a specific design in order to achieve desired function with acceptable performance, the types of components and their configuration have a directly effect on the system reliability. So the parallel system, which is one of the most common forms of redundancy for increasing reliability can successfully operate as long as one of the units is working and it fails if all it's units fail.

2. Methodology

In this section, parallel system reliability, Markov chain, the two state Markov process, the mean time to failure and the finite Markov chain with recurrent and transient states are defined.

2.1. Parallel System Reliability

The system reliability function of parallel units is defined as the probability that not all the parallel units fail in a time interval given that all units are operating at the beginning of the interval suppose that a parallel system consists of (n) independent units with the life times $T_1, T_2, \dots, T_n$, then the life time of the system T is defined as

$$T = max(T_1, \dots, T_n)$$

Thus, the system reliability at t is

$$\begin{aligned} R(t) &= P_r(T > t) \\ &= P_r[max(T_1, \dots, T_n) > t] \\ &= 1 - \pi_{i=1}^{n}(1 - R_i(t)) \end{aligned}$$

Where $0 \le R_i(t) \le 1$ for all $i = 1,2, \dots, n$

And the mean time to failure (MTTF) of the system is given by

$$E(T) = \int_0^\infty R(t)dt$$

2.2. Markov chain

Markov processes with discrete parameter (time) and discrete state spaces , called a Markov chain $\{x_n \,;\, n = 0,1,2, \dots\}$, if

$$\begin{aligned} p_{ij} &= p_r(x_n = j | x_{n-1} = i \,, x_{n-2} = i_1 \,, \dots \,, x_0 = i_{n-1}) \\ &= p_r(x_n = j | x_{n-1} = i) \end{aligned}$$

and

$$p_{ij}^n = p_r(x_n = j | x_0 = i)$$

For the m-state Markov chain with the transition probability matrix (p_{ij}) and the initial probability distribution , denoted by a vector (π_i) , where

$$\sum_{j=0}^{m-1} p_{ij} = 1 \quad and \quad \sum_{i=0}^{m-1} \pi_i = 1 \qquad for\ each\ (i)$$

A stationary probability distribution (π_j) of Markov chain is defined as

$$\pi_j = \sum_{i=0}^{m-1} \pi_i p_{ij} \qquad for\ all\ j$$

Also for a finite ergodic Markov chain

$$\lim_{n \to \infty} p_{ij}^n = \pi_j$$

$$\lim_{n\to\infty} \pi_i^n = \pi_j$$

2.3. The two state Markov Process

The most common assumption about the statistical characteristics of failures is that they form a poisson process so the failure time and repair time have independent exponential distributions with parameters λ *and* μ respectively. Let the process $\{X(t); t \geq 0\}$ with states $\{0,1\}$, in which a failure can occur when the process is in state (0) and a repair can occur only when the process is in state (1), thus the state change are from (0) to (1) with failure rate λ and from (1) to (0) with repair rate μ. The transition probabilities of this process are defined as:

$$P_{ij}(t) = P(X(t) = j|X(0) = i) \quad i,j = 0,1$$

$$P_{00}(t) = \frac{\mu + \lambda e^{-(\lambda+\mu)t}}{\lambda + \mu}$$

........1

$$P_{10}(t) = \frac{\mu - \mu e^{-(\lambda+\mu)t}}{\lambda + \mu}$$

$$P_{01}(t) = 1 - P_{00}(t) \quad , \quad P_{11}(t) = 1 - P_{10}(t)$$

Also the stationary distribution, (as $t \to \infty$) are

$$\lim P_{00}(t) = \lim P_{10}(t) = \frac{\mu}{\lambda + \mu}$$

and 2

$$\lim P_{01}(t) = \lim P_{11}(t) = \frac{\lambda}{\lambda + \mu}$$

2.4. Mean time to failure (MTTF) by probability assumptions

In parallel system, the system operates iff at least one of the n units operates. So a system which has two identical parallel units with failure rate λ and repair rate (μ), being in one of the following states:

State (0) : both units operative

State (1) : only one unit operative, the other having failed

State (2) : both units have failed

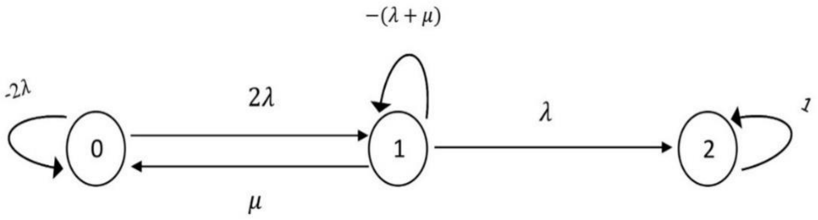
Then, the probabilities of fail and repair an unit during $(t, t+h)$ are given as the following:

i. The probability that one unit fail is $(\lambda h + 0(h))$ and it does not fail with probability $(1 - \lambda h + 0(h))$.
ii. The probability that a failure unit repaired is $(\mu h + 0(h))$ and it does not repaired with probability $(1 - \mu h + 0(h))$.
iii. All other probabilities are $0(h)$, such that $\frac{0(h)}{h} = 0$ *as* $h \to 0$ These transition probabilities lead to the following generator matrix

$$\begin{pmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 0 & 1 \end{pmatrix}$$

……..3

and it's transition diagram



Let $P_1(t)$ be the probability that the system in state i $(i = 0,1,2)$ at time (t) with initial conditions $P_0(0) = 1$, $P_2(\infty) = 1$. Then

$$P_0(t+h) = P_0(t) * [(1-\lambda h)(1-\lambda h)] + P_1(t)(1-\lambda h)\mu h + 0(h)$$

$$P_1(t+h) = P_0(t) * 2\lambda h + P_1(t)(1-\lambda h)(1-\mu h) + 0(h)$$

$$P_2(t+h) = P_1(t)\lambda h + P_2(t) + 0(h)$$

$$P_0'(t) = -2\lambda P_0(t) + \mu P_1(t)$$

$$P_1'(t) = 2\lambda P_0(t) - (\lambda+\mu)P_1(t)$$

$$P_2'(t) = \lambda P_1(t)$$

by integrating the equations and substitution the initial conditions, we have

$$-1 = -2\lambda \int_0^\infty P_0(t)dt + \mu \int_0^\infty P_1(t)dt$$

$$0 = 2\lambda \int_0^\infty P_0(t)dt - (\lambda+\mu) \int_0^\infty P_1(t)dt$$

$$1 = \lambda \int_0^\infty P_1(t)dt$$

$$\int_0^\infty P_1(t)dt = \frac{1}{\lambda} \qquad and \quad \int_0^\infty P_0(t)dt = \frac{\lambda+\mu}{2\lambda^2}$$

Since MTTF $= \int_0^\infty P_0(t)dt + \int_0^\infty P_1(t)dt$

Then MTTF $= \frac{\lambda+\mu}{2\lambda^2} + \frac{1}{\lambda} = \frac{3\lambda+\mu}{2\lambda^2}$ …….4

2.5. Finite Markov chain with recurrent and transient states

For m-state Markov chain with (r) recurrent states and (m-r) transient states, the transition probability matrix P can be represent as in the following submatrices

$$P = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1m} \\ \vdots & \vdots & \vdots & \vdots \\ P_{m1} & P_{m2} & \dots & Pmm \end{pmatrix}$$

can be represent as in the following submatrices

$$P = \begin{pmatrix} P^* & O \\ R & Q \end{pmatrix} \qquad ....5$$

Where

$P^*_{r\times r}$:define the transition probabilities among the recurrent states.

$Q_{(m-r)\times(m-r)}$:define the transition probabilities among the transient states.

$R_{(m-r)\times r}$: define the transition probabilities from the (m-r) transient states to the (r) recurrent states.

$O_{r\times(m-r)}$: is zero submatrix .

Then, the mean number of transitions (M) between transient states before entering a recurrent state is defined as

$$M = (I - Q)^{-1} \qquad ....6$$

$$= \begin{pmatrix} \mu_{r+1,r+1} & \mu_{r+1,r+2} & \cdots & \mu_{r+1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m,r+1} & \mu_{m,r+2} & \cdots & \mu_{m,m} \end{pmatrix}$$

and the mean number of steps (M_s) that the process required to leave transient states and it eventually enters any one of the recurrent states is

$$M_s = \left(\sum_{j \epsilon T} \mu_{ij} \right) \qquad i, j\epsilon \; transient \; states \; (T)$$

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$$= \begin{pmatrix} \sum_{j\epsilon T} \mu_{r+1,j} \\ \sum_{j\epsilon T} \mu_{r+2,j} \\ . \\ . \\ . \\ \sum_{j\epsilon T} \mu_{m,j} \end{pmatrix} \qquad , \qquad j = r+1, r+2, \ldots \; , m$$

and the transition probabilities (f_{ij}) from transient state (i), the process eventually enters a given recurrent state j are defined as

$$F = MR.$$
$$= (fij) \qquad ....8$$

3. Application

This section specified for application, which includes computation of the transition probabilities of the two state Markov process, the mean time to failure of a two identical unit parallel system by the probability assumptions and by the Markov chain with recurrent and transient states.

3.1. Data collection

The data about the failure and repair times of a system composed two identical electric generators connected in parallel of the months november and december in 2022 are collected from the electricians of 10 different places in Erbil city, whereas they supplied electricity for (8) hours daily to their lanes. From the available data, the approximate weekly failure rate is 2 hours and repair rate is 0.5 hour, are found.

So the per hour failure rate is

$$\lambda = \frac{2}{7 \times 8} = 0.0714$$

And the per hour repair rate is

$$\mu = \frac{0.5}{7 \times 8} = 0.009$$

3.2. Determination the transition probabilities of the two state Markov process

The transition probabilities of the two state Markov process $\{0,1\}$ during 8 hours are determined eq(1) as in the following table

time	P_{00}	P_{01}	P_{10}	P_{11}
1	0.964797985	0.035202015	0.008800504	0.991199496
2	0.931144948	0.068855052	0.017213763	0.982786237
3	0.898972729	0.101027271	0.025256818	0.974743182
4	0.868216169	0.131783831	0.032945958	0.967054042
5	0.838812975	0.161187025	0.040296756	0.959703244
6	0.810703595	0.189296405	0.047324101	0.952675899
7	0.783831099	0.216168901	0.054042225	0.945957775
8	0.758141061	0.241858939	0.060464735	0.939535265

Table (1) shows the transition probabilities with time.

3.3. Representation the transition probability matrix of a finite Markov chain with recurrent and transient states

The transition probability matrix P of the continuous time Markov chain with two transient states $\{0,1\}$ and one recurrent state (2) has the following form, eq(3)

$$P = \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 0 & 1 \end{pmatrix}$$

For $\lambda = 0.0714/h$ and $\mu = 0.009/h$, we get

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 0.9286 & 0.0714 & 0 \\ 0.009 & 0.9553 & 0.0357 \\ 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

This, the transition probability matrix P can put in canonical form as, eq(5)

$$P = \begin{matrix} & \begin{matrix} 2 & 0 & 1 \end{matrix} \\ \begin{matrix} 2 \\ 0 \\ 1 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.9286 & 0.0714 \\ 0.0357 & 0.009 & 0.9553 \end{pmatrix} \end{matrix}$$

3.3.1. Define the mean number of transitions among transient states

The mean number of transitions (M) the process makes between transient states {0,1} eq(6), is defined as

$$M = (I - Q)^{-1}$$

$$= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.9286 & 0.0714 \\ 0.009 & 0.9553 \end{pmatrix}\right]^{-1} = \begin{bmatrix} 0.0714 & -0.0714 \\ -0.009 & 0.0447 \end{bmatrix}^{-1}$$

$$= \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{pmatrix} 17.53 & 28 \\ 3.53 & 28 \end{pmatrix} \end{matrix} = [\mu_{ij}]$$

and the mean number of steps (M_s) the process spends in transient states {0,1} before entering a recurrent state (2) is

$$M_s = \begin{matrix} 0 \\ 1 \end{matrix} \begin{pmatrix} 45.53 \\ 31.53 \end{pmatrix}$$

3.3.2. Define the transition Probabilities from transient states to recurrent state

The probability that the process starts from transient states {0,1} and enters a recurrent state {2} eq(8), is defined as

$$F = \begin{pmatrix} 17.53 & 28 \\ 3.53 & 28 \end{pmatrix} \begin{pmatrix} 0 \\ 0.0357 \end{pmatrix}$$

$$2$$

$$= \begin{matrix} 0 \\ 1 \end{matrix} \begin{pmatrix} 0.9996 \\ 0.9996 \end{pmatrix}$$

3.4. Determine the mean time to system failure (MTSF)

The mean time to system failure is determined by the following two methods:

i. Probability assumptions

$$MTSF = \frac{3\lambda + \mu}{2\lambda^2} = \frac{3 \times 0.0357 + 0.009}{2(0.0357)^2} = 45.53$$

ii. Continuous time Markov chain with recurrent and transient states

By this method the mean time to system failure is equal to the mean number of steps the process spent in state {0} and the mean number of steps the process transmits from state {0} to state {1} eq(7)

$$MTSF = \mu_{00} + \mu_{01}$$

$$= 17.53 + 28$$

$$= 45.53$$

Also by eq(2), the probability that a system is working for along time π_0 is

$$\pi_0 = \frac{0.009}{0.0357 + 0.009} = 0.2$$

and the probability that it fails π_1 is

$$\pi_1 = \frac{0.0357}{0.0357 + 0.009} = 0.8$$

4. Conclusions

From the results of application the following conclusions are found

1. The probabilities of generator fails and repairs are increasing with time.
2. The Markov chain with recurrent and transient states is most suitable method to determine the mean time to failure a system composed of more than two parallel units.
3. The mean time to system failure by Markovian method is equal to the mean number of steps the process spends in transient states.
4. The mean time to system failure by the two methods are equal.
5. By Markovian method system failure does occur, when the process leaves transient states and enters a recurrent state.
6. The stationary distribution of the two state Markov process shows that a system operates with lowest probability and it fails with highest probability.

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تحليل عشوائي لحساب الوقت لفشل النظام (MTSF) لنظام متوازي موحد متطابق

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ملخص

في هذا البحث تم تحليل مولدين كهربائيين متوازيين متطابقين مع إصلاح واحد فاشل عشوائيا بطريقتين ، الأولى بالطريقة بناء على افتراضات احتمالية معينة والثانية من خلال سلسلة ماركوف ثلاثية الحالة ، مصنفة على أنها حالة متكررة (امتصاص) وحالتين عابرتين. ولغرض هذه الدراسة يتم الحصول على البيانات حول الفترات الزمنية لتعطل وإصلاح المولدات الكهربائية لشهري تشرين الثاني وكانون الأول 2022 من كهربائيي عدة أماكن في مدينة أربيل، حيث قاموا بتزويد مساراتهم بالكهرباء كلما انقطعت الطاقة الكهربائية الوطنية. الهدف الرئيسي من هذا البحث هو تحديد متوسط الوقت لفشل النظام بالطريقتين السابقتين.

الكلمات المفتاحية: نظام التوازي مع الاصلاح ، متوسط وقت الفشل ، مصفوفة الاحتمالات الانتقالية.

شيكردنەوەيەكى هەرەمەكى بۆ هەژماركردنى ماوەى كاتى شكستى سيستەم (MTSF) لە دوو سيستەمى هاوتەريبى يەكسان

بێستون ميرزا عبدالكريم

بەشى ئامار،كۆلێژى بەڕێوەبردن و ئابوورى ،زانكۆى سەلاحەدين - هەولێر

پوختە

لەم توێژینەوەیەدا دوو جێنەرەیتەری کارەبایی هاوتەریبی هاوشێوە لەگەڵ چاککردنەوەی یەکێکی شکستخواردوو شیکردنەوەیان بۆ کراوە بە دوو ڕێگا، یەکەم بە ڕێگا لەسەر بنەمای گریمانەی ئەگەری دیاریکراو و دووەم لە ڕێگەی سێ زنجیرەی مارکۆڤ کە پۆلێن کراوە وەک یەک دووبارەبوونەوە (مژین) و دوو دۆخی کاتی. بۆ مەبەستی ئەم لێکۆڵینەوەیە، داتاکان دەربارەی شکست و کاتی چاککردنەوەی مۆلیدە کارەباییەکانی مانگەکانی تشرینی دووەم و کانوونی یەکەمی 2022 لە کارەبایی چەند شوێنێکی شاری هەولێرەوە دەست دەکەوێت، لە کاتێکدا کارەبا بۆ ڕێڕەوەکانیان دابین دەکەن هەر کاتێک کارەبای نیشتمانی پچڕا. ئامانجی سەرەکی ئەم توێژینەوەیە ئەوەیە کە کاتی شکستی سیستەمەکە دیاری بکات بە دوو ڕێگای پێشوو.

وشە سەرتاییەکان: سیستەمی هاووتەریبوون و جاك کردنەوە ، ماوەی کاتی شکست بوون ، ڕێزکراوەکانی ئەگەری گواستنەوەکان.